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RADIAL AIRCRAFT-ENGINE BEARING LOADS

I - CRANKPIN-BEARING LOADS FOR ENGINES HAVING

NINE CYLINDERS PER CRANKPIN

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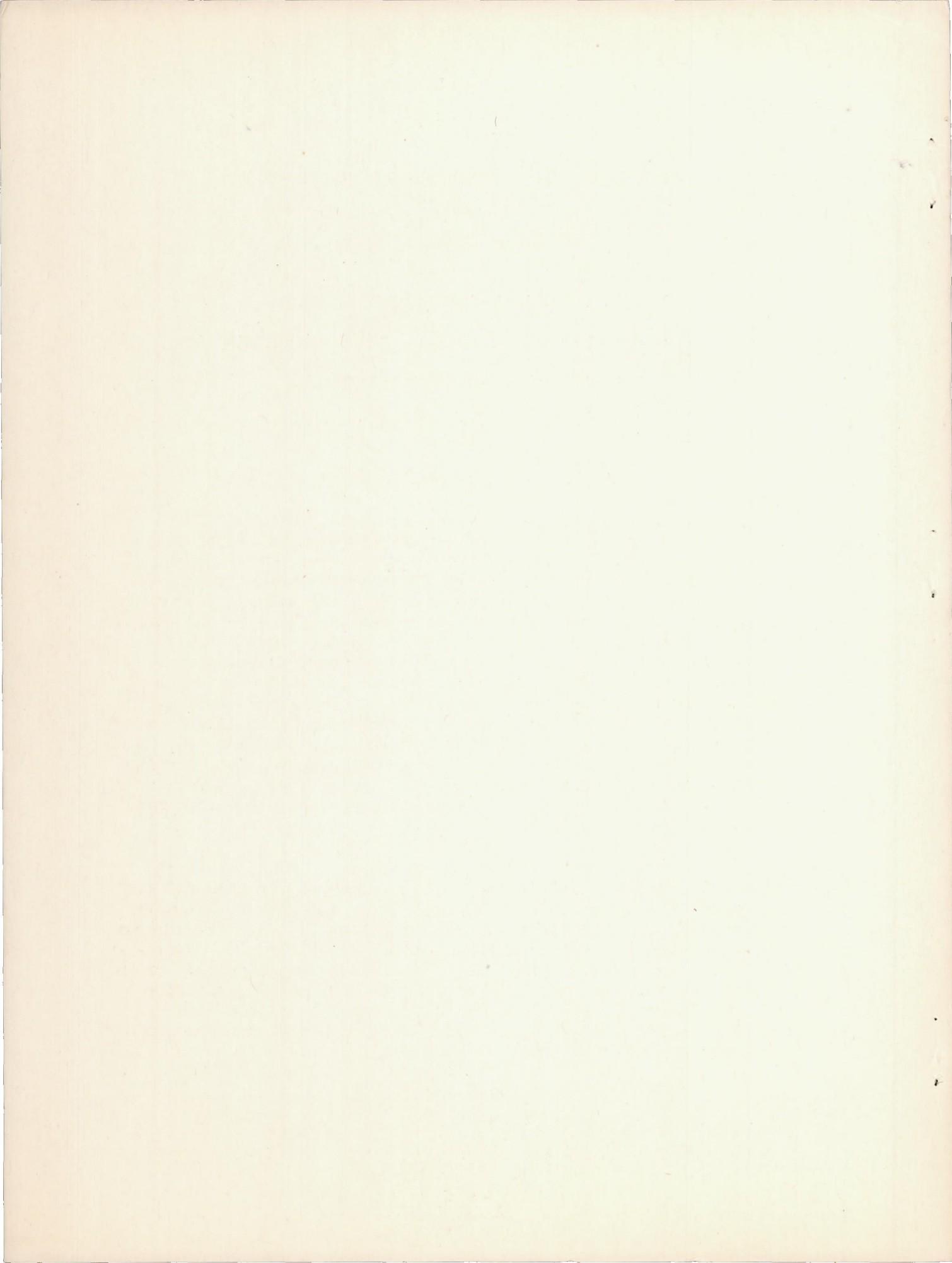
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ADVANCE RESTRICTED REPORT

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SUMMARY

Dimensional analysis and the principle of similitude are applied to the computation of the crankpin-bearing loads of radial engines having nine cylinders per crankpin. A method of generalizing the results of a relatively few conventional load analyses is developed to determine crankpin-bearing loads under a wide range of operating conditions. Charts are presented that give the maximum and mean crankpin-bearing loads for a production engine at all values of engine speed to 5000 rpm and at all values of indicated mean effective pressure to 500 pounds per square inch. By use of speed and load correction factors these charts may be readily applied to all engines having nine cylinders per crankpin, and to illustrate this extended use two other production engines are considered. The individual effects of the several engine dimensions (reciprocating weight, rotating weight, connecting-rod length, stroke, bore, and compression ratio) upon the crankpin-bearing loads are determined and discussed.

It was found that optimum combinations of engine speed and indicated mean effective pressure exist for which the mean and maximum crankpin-bearing loads are minima for a given power, but such combinations lie in an impracticable operating region. The maximum and mean crankpin-bearing loads in the practicable region of operation are decreased by an increase of indicated mean effective pressure or compression ratio. Polar diagrams of bearing loads show that the shock load and the range of stress imposed on a crankpin bearing increases with increased indicated mean effective pressure and compression ratio.

INTRODUCTION

The influence of indicated mean effective pressure and engine speed on the maximum and mean loads acting upon the principal bearings of an aircraft engine is important in both the design and the operation of the engine. In addition, the effects of recipro-cating or rotating weight, connecting-rod length, stroke, bore, and compression ratio on the bearing loads are of interest.

In 1919 Burkhardt (references 1 and 2) devised a method of computing internal-combustion-engine bearing loads. This method was applied to a crankpin bearing of a radial aircraft engine by Prescott and Poole (reference 3) in 1931. Inasmuch as the conven-tional computation of bearing loads is tedious and time consuming, several approximate solutions have been proposed (references 3 to 5), but none of these methods are entirely satisfactory.

A scheme for generalizing a relatively few conventional bearing-load analyses has been applied to an in-line engine. (See reference 6.) With this method, a few bearing-load computations may be extended to obtain charts that give the mean and maximum bearing load at any combination of engine speed and indicated mean effective pressure.

This method has been applied to the radial-type engine to illustrate further the generalized treatment and load charts are presented herein for a production engine, which will be designated engine A. In order to demonstrate the applicability of the charts to other radial engines having nine cylinders per crankpin, two other production engines, designated engines B and C, are also considered.

THEORY

The dimensional method of reasoning presented by Buckingham (reference 7) has been applied to the generalization of V-type engine bearing loads in reference 6. The significant variables that may affect a radial-engine bearing load are:

Symbol	Variable	Dimensional formula	Relation
N	Engine speed, rpm	T^{-1}	Independent
p	Indicated mean effective pressure, pounds per square inch	FL^{-2}	Do.
L_S	Stroke, inches	L	Do.
W	Crankpin bearing load, pounds	F	π_a
M_i	Reciprocating mass per crankpin, slugs	FT^2L^{-1}	π_b
M_c	Rotating mass per crankpin, slugs	FT^2L^{-1}	π_c
D	Diameter of bore, inches	L	π_d
L_R	Length of master connecting-rod length, inches	L	π_e
p_m	Manifold pressure, pounds per square inch absolute	FL^{-2}	π_f
r	Compression ratio	None	π_g
θ	Crank angle, degrees	----do-------	π_h
n	Number of equally spaced cylinders	----do-------	π_i

The notation used throughout the report is recapitulated in appendix A.

If the engine speed, the indicated mean effective pressure, and the stroke are taken as the independent variables, an application of Buckingham's π theorem (reference 8) yields the following equation:

$$W = L_S^2 \, p \, \Omega\left(\frac{M_i N^2}{L_S \, p}, \; \frac{M_c N^2}{L_S \, p}, \; \frac{D}{L_S}, \; \frac{L_R}{L_S}, \; \frac{p_m}{p}, \; \theta, \; r, \; n\right) \qquad (1)$$

where Ω is some function of the nondimensional quantities within parentheses.

A similar equation may be derived for the direction of the resultant bearing load. In addition to N, p, L_S, M_i, M_c, D, L_R, p_m, r, θ, and n, the direction of the resultant bearing load is affected by σ, the angle (in deg) between the axis of cylinder 1 and the resultant bearing load in the direction of rotation.

When engine speed, indicated mean effective pressure, and mean stroke are taken as independent variables, and Buckingham's π theorem is applied, the following equation is obtained:

$$\sigma = \Omega' \left(\frac{M_i}{L} \frac{N^2}{p}, \frac{M_c}{L} \frac{N^2}{p}, \frac{D}{L_S}, \frac{L_R}{L_S}, \frac{p_m}{p}, r, \theta, n \right) \qquad (2)$$

where Ω' is some function of the nondimensional quantities within parentheses.

If the indicated mean effective pressure is assumed to be proportional to the manifold pressure, equations (1) and (2) simplify to the following expressions for a specific engine:

$$W = p \, \Omega'' \left(\frac{N^2}{p}, \theta \right) \qquad (3)$$

$$\sigma = \Omega''' \left(\frac{N^2}{p}, \theta \right) \qquad (4)$$

Equation (3) establishes the fact that, if W/p is plotted against N^2/p at a constant value of crank angle, a smooth curve will be obtained. Similarly, equation (4) states that, at a particular crank angle, a specific relation exists between the angle σ and N^2/p.

Prescott and Poole (reference 3) state that the effect of articulation of a radial engine may be neglected in computing bearing loads and all connecting rods may be considered to intersect the center of the crankpin. The inertia and gas forces at all cylinders are also assumed to be equal. They cite the fact that an exact analysis, which considered articulation and the small differences that exist from cylinder to cylinder, took one man 9 months to complete and did not differ from the equivalent simplified conventional analysis, which may be made in from 2 to 3 days.

The following equation from reference 3 (with a change of notation) is an expression for the resultant inertia force acting on the crankpin of a radial engine having five, seven, or nine cylinders:

$$W_r = 28.4 \times 10^{-6} \frac{L_S}{2} N^2 \left\{ F_c + \left[\frac{1}{2} + \frac{1}{4} \left(\frac{L_S}{2\ L_R} \right)^2 \right] F_i \right\} \qquad (5)$$

where

W_r resultant inertia force due to all rotating and reciprocating mass that acts upon crankpin, pounds

F_c rotating weight per crankpin (contributing to centrifugal force), pounds

F_i reciprocating weight per crankpin (contributing to inertia force), pounds

This resultant inertia force is also reported to act with the same direction and sense as the corresponding rotating-weight component (that is, radially outward along the crank axis). Equation (5), which is developed in appendix B, is important in that its use considerably simplifies the computation of crankpin-bearing loads and it is a good approximation for radial engines having 5, 7, 9, or 11 cylinders.

The method used in this report utilizes equations (3), (4), and (5). A polar diagram of resultant gas force is first determined, for any convenient indicated mean effective pressure, in the usual manner by adding vectorially the gas forces of the individual cylinders. Because of the inherent symmetry of the radial-type engine, only the crank-angle interval from $0°$ to $720°/n$ need be considered and, therefore, in this report only those events occurring at crank-angle values from $0°$ to $80°$ are investigated. The resultant inertia-force circles corresponding to a number of engine speeds are then obtained from equation (5). Curves of resultant bearing load are obtained by the vector addition of corresponding resultant gas-force and resultant inertia-force components. Equations (3) and (4) may be used to plot W/p against N^2/p and σ against N^2/p for each value of crank angle investigated. These plots are applicable not only to the value of indicated mean effective pressure for which they were derived but to all values of indicated mean effective pressure. The mean load may be generalized in the same manner from determinations made at a single value of indicated mean effective pressure.

APPLICATION OF THE DIMENSIONAL-ANALYSIS METHOD TO ENGINE A

Conventional Computation of Crankpin-Bearing Loads

The symbols and conventions used in this application are defined in the preceding section and in figure 1 and are recapitulated in appendix A. Specifications for engine A are given in appendix C, and sketches of the connecting-rod and the crankshaft arrangement are shown in figures 2 and 3, respectively. Throughout this report, a crank angle θ of 0° refers to the top-center position of the piston in cylinder 1 at the beginning of the expansion stroke.

The indicated mean effective pressure corresponding to rated take-off power (245 lb/sq in.) was employed in this analysis. The standard cycle-indicator diagram for this condition (fig. 4) was constructed according to reference 3, using exponents of 1.30 for both the expansion and the compression curves, a diagram factor of 0.90, and a maximum gas pressure equal to 75 percent of the computed maximum.

The gas force at any value of crank angle is equal to the product of the corresponding value of the pressure from the indicator diagram and the piston area. The reciprocating inertia force at any crank angle is equal to the product of the reciprocating mass and the piston acceleration. Values of acceleration may be found in Smith's compilation of piston accelerations (reference 9). The resultant load acting on the piston pin parallel to the cylinder axis is obtained by algebraically adding the gas force and the reciprocating inertia force. This resultant load, when multiplied by the secant of the angle ϕ (fig. 1) gives the force acting along the connecting-rod axis.

The centrifugal force acting on the crankpin may be computed from the values of the rotating mass, the crank throw, and the engine speed. By the method given in reference 3 the resultant load acting on the crankpin at any particular crank angle is obtained by the vector addition of the centrifugal force and the nine forces acting along the cylinder axes (each of which is composed of the algebraic sum of the gas and reciprocating inertia forces) with account taken of the firing order of the engine. Such vectors constituting the resultant crankpin bearing load OD at a crank angle of 20° are labeled OA and numbered 1 to 9 in figure 5, which is a representative polar diagram of forces acting on the crankpin with respect to the engine axis for an engine speed of 2500 rpm and an indicated mean effective pressure of 245 pounds per square inch

In the application of the method of this report it is more
convenient to determine the resultant of the nine individual gas-
force components and add this vector to the resultant inertia force
vector (from equation (5)). This method is also illustrated in fig-
ure 5 for a crank angle of 20° where OB is the resultant inertia-
force vector, OC is the resultant gas-force vector (composed of
the nine individual components numbered 1' to 9'), and OD is again
the resultant load on the crankpin obtained by the vector addition
of vectors OB and OC. The resultant gas-force curve and the
rotating inertia-force and resultant inertia-force circles are also
shown in figure 5.

Generalized Load Charts

Maximum bearing loads. - Resultant inertia-force circles were
constructed from equation (5) for a number of speeds corresponding
to values of N^2/p from 0 to 50,000. Resultant bearing-load
vectors were obtained by adding vectorially values of resultant gas
force for an indicated mean effective pressure of 245 pounds per
square inch (from fig. 5) to the corresponding resultant total
inertia-force vectors (from equation (5)). Such data are plotted
in figures 6 and 7 according to equations (3) and (4) with crank
angle as the parameter. The solid portions of the curves (for
crank-angle values of 20° and 50°) of figure 6 correspond to the
maximum value of W/p over the entire range of N^2/p shown.

A useful chart (fig. 8) for determining maximum crankpin loads
is obtained from the curves of figure 6. The line OA represents
the locus of optimum combinations of speed and indicated mean effec-
tive pressure for which the maximum bearing load at a given power
level is a minimum.

The bearing loads determined from the maximum load chart for
the conditions of (1) take-off, (2) take-off engine speed and
10 percent above take-off indicated mean effective pressure, and
(3) take-off indicated mean effective pressure and 10 percent above
take-off engine speed are given in the following table:

Condi-tion	Indicated mean effec-tive pres-sure (lb/sq in.)	Engine speed (rpm)	Maximum bearing load (lb)	Maximum unit bearing load[a] (lb/sq in.)
1	245	2500	39,500	3920
2	270	2500	39,000	3860
3	245	2750	48,500	4800

[a]The effective projected bearing area was taken
as 10.1 sq in.

Mean bearing loads. - The mean load $\overline{W}$ acting on the crankpin bearing was determined by plotting W against crank angle, using a planimeter to obtain the average height of the curve, and was used to obtain figure 9 in which $\overline{W}/p$ is plotted against N^2/p.

A convenient chart (fig. 10) is obtained from figure 9 by plotting indicated mean effective pressure against engine speed with mean bearing load as the parameter. The use of the mean load chart is illustrated in the following example. The bearing loads for the previously tabulated three conditions are as follows:

Condition	Indicated mean effective pressure (lb/sq in.)	Engine speed (rpm)	Mean bearing load (lb)	Mean unit bearing load[a] (lb/sq in.)
1	245	2500	33,000	3270
2	270	2500	31,500	3140
3	245	2750	41,500	4110

[a]The effective projected bearing area was taken as 10.1 sq in.

Polar diagrams. - A polar diagram of the resultant crankpin-bearing load may be obtained for any combination of engine speed and indicated mean effective pressure from the curves of figures 6 and 7. Representative polar diagrams with respect to the engine axis (fig. 11) have been constructed for the following four power conditions:

Indicated mean effective pressure (lb/sq in.)	Engine speed (rpm)	Indicated horsepower
0	2500	0
150	2500	860
250	2500	1440
350	2500	2020

Inasmuch as these diagrams are similar for each span of $720°/n$, they are shown only from $710°$ to $80°$ crank angle. A polar diagram with respect to the crank axis is more useful than a diagram with respect to the engine axis in defining loads acting on the crankpin.

Polar diagrams with respect to the crank axis may be obtained by rotating each resultant vector of figure 11 counter to the direction of rotation through an angle corresponding to the number of crank-angle degrees indicated at the terminal end of the vector.

The polar diagrams with respect to the crank axis for the four power conditions of the foregoing table are given in figure 12 in terms of crank-angle degrees. This figure enables the polar diagram for any engine speed and indicated mean effective pressure to be visualized. Point A is the entire polar diagram with respect to the crank axis for an indicated mean effective pressure of 0 pounds per square inch. The distance from point A to the pole corresponding to any engine speed is obtained from equation (5). The upper and lower envelopes of the polar diagrams for different indicated mean effective pressures intersect at point A. The pole corresponding to any engine speed is located along the crank axis by projecting horizontally from the speed scale. For example, line OB represents to scale the magnitude and direction of the force on the crankpin of engine A with respect to the crank axis at an engine speed of 2000 rpm, an indicated mean effective pressure of 250 pounds per square inch, and a crank angle of 20°. The speed scales for engines B and C will be discussed under APPLICATION OF THE DIMENSIONAL ANALYSIS METHOD TO OTHER ENGINES HAVING NINE CYLINDERS PER CRANKPIN.

Figure 13 shows that the polar diagrams with respect to the crank axis for a particular speed and indicated mean effective pressure may be obtained from a diagram for the same indicated mean effective pressure but a different engine speed by translating the center of the crankpin (the pole) along the crank axis. Portions of two polar diagrams with respect to the engine axis for the same indicated mean effective pressure but different engine speeds are shown in this figure. Line BC is parallel to line AD and, when these diagrams are rotated to obtain diagrams with respect to the crank axis, line BC will rotate into line B'C'. It is thus evident that the diagram with respect to the crank axis is merely translated in a direction parallel to this axis when the engine speed varies and the indicated mean effective pressure is constant.

The polar diagrams with respect to the master connecting-rod axis are also of interest with regard to loads acting on the bearing surface. These diagrams are obtained by rotating the diagram with respect to the crank axis in the direction of crankshaft rotation through an angle of $180 + \alpha_1$, where α_1 is the angle defined in figure 1. Polar diagrams with respect to the master connecting-rod axis for an engine speed of 2500 rpm and indicated mean effective pressures of 0 and 350 pounds per square inch are given in figure 14

in terms of crank-angle degrees. The relative velocity between the bearing and the crankpin destroys the symmetry found in the polar diagram with respect to the engine axis (fig. 5).

Rubbing factor. - The rubbing factor (reference 3) may be obtained by inserting the mean load from figure 10 into the equation

$$RF = (1.41 \times 10^{-3}) \, N\overline{W} \tag{6}$$

where RF is the rubbing factor, (ft-lb)/(sq in.)(sec). Although the rubbing factor is not generally considered a good criterion for the severity of bearing operating conditions, it is given for convenience.

Verification of the Generalized Load Charts

In order to check the generalized load charts (figs. 6, 7, 8, and 10), a conventional analysis was made for an engine speed of 2500 rpm and an indicated mean effective pressure of 368 pounds per square inch. The results obtained from this analysis and from the generalized load charts are tabulated:

Crank angle (deg)	From conventional analysis		From figures 6 and 7		Percentage difference	
	W (lb)	σ (deg)	W (lb)	σ (deg)	W	σ
0	29,000	18.5	28,500	18.5	1.7	0
10	17,000	70.5	16,500	70.0	2.9	.7
20	24,000	89.5	24,000	90.0	0	.5
30	30,500	79.0	30,500	79.0	0	0
40	35,500	75.0	35,500	75.0	0	0
50	38,500	74.5	37,500	73.0	2.6	2.0
60	36,000	78.0	36,000	77.5	0	.6
70	32,500	86.0	32,500	85.5	0	.6
Mean	30,000	-------	30,000	------	0	----

This close agreement is considered a satisfactory check of the accuracy of the computations as well as of the generalization method.

Individual Effects of Engine Dimensions upon Crankpin-Bearing Loads

Reciprocating weight. - It is evident from equation (1) that an increase in reciprocating weight may be compensated by a decrease in engine speed. Crankpin-bearing loads corresponding to any reciprocating weight at any value of crank angle may thus be determined from figures 6, 7, and 9 by use of an equivalent engine speed.

From equation (5):

$$W_r = 28.4 \times 10^{-6} \frac{L_S}{2} N^2 \left\{ F_c + \left[\frac{1}{2} + \frac{1}{4} \left(\frac{L_S}{2 \ L_R} \right)^2 \right] F_i' \right\} \qquad (7)$$

$$W_r = 28.4 \times 10^{-6} \frac{L_S}{2} N_i^2 \left\{ F_c + \left[\frac{1}{2} + \frac{1}{4} \left(\frac{L_S}{2 \ L_R} \right)^2 \right] F_i \right\} \qquad (8)$$

where

F_i standard reciprocating weight per crankpin, pounds

F_i' new reciprocating weight per crankpin, pounds

N_i equivalent engine speed with standard reciprocating weight, rpm

N actual engine speed with new reciprocating weight, rpm

Equating (7) and (8) and solving for N_i:

$$N_i = \left\{ \frac{F_c + \left[\frac{1}{2} + \frac{1}{4} \left(\frac{L_S}{2 \ L_R} \right)^2 \right] F_i'}{F_c + \left[\frac{1}{2} + \frac{1}{4} \left(\frac{L_S}{2 \ L_R} \right)^2 \right] F_i} \right\}^{1/2} N \qquad (9)$$

Because $L_S/2 \ L_R$ is equal to 0.25 for engine A,

$$N_i = \left(\frac{F_c + 0.516 \ F_i'}{F_c + 0.516 \ F_i} \right)^{1/2} N \qquad (10)$$

The maximum and mean crankpin-bearing loads for any engine speed N, indicated mean effective pressure p, and the new reciprocating weight F_i' may be determined from figures 8 and 10, by use of the actual indicated mean effective pressure and the equivalent engine speed N_i from equation (10).

The relative effect of a 10-percent increase or decrease in the reciprocating weight per crankpin upon maximum and mean bearing loads is illustrated for take-off conditions in the following table, which was obtained from equation (10) and figures 8 and 10:

Standard reciprocating weight F_i		10-percent increase in reciprocating weight $F_i' = 1.1\,F_i$				10-percent decrease in reciprocating weight $F_i' = 0.9\,F_i$			
W	$\overline{W}$	W		$\overline{W}$		W		$\overline{W}$	
(lb)	(lb)	(lb)	(percent)	(lb)	(percent)	(lb)	(percent)	(lb)	(percent)
39,500	33,000	42,000	6.3	35,000	6.1	37,000	-6.3	30,000	-9.1

Rotating weight. - The crankpin-bearing load corresponding to any rotating weight at any value of crank angle may also be determined from figures 6, 7, and 9 by use of an equivalent engine speed. This equivalent engine speed may be obtained in a manner similar to that used to derive equation (10)

$$N_c = \left(\frac{F_c' + 0.516\,F_i}{F_c + 0.516\,F_i}\right)^{1/2} N \qquad (11)$$

where

F_c standard rotating weight per crankpin, pounds

F_c' new rotating weight per crankpin, pounds

N_c equivalent engine speed with standard rotating weight, rpm

N actual engine speed with new rotating weight, rpm

The relative influence of a 10-percent increase or a 10-percent decrease of the rotating weight per crankpin upon the maximum and mean bearing loads is illustrated for take-off conditions (2500 rpm, 245 lb/sq in.) in the following table, which was obtained from equation (11) and figures 8 and 10:

Standard rotating weight F_c		10-percent increase in rotating weight $F_c' = 1.1 F_c$				10-percent decrease in rotating weight $F_c' = 0.9 F_c$			
W	$\overline{W}$	W		$\overline{W}$		W		$\overline{W}$	
(lb)	(lb)	(lb)	(percent)	(lb)	(percent)	(lb)	(percent)	(lb)	(percent)
39,500	33,000	41,500	5.1	34,000	3.3	37,500	-5.1	30,500	-7.6

Connecting-rod length. - The connecting-rod length plays an unimportant part in the development of bearing loads. It affects only the ratio of the connecting-rod length to the crank throw which, in turn, influences the acceleration of the piston and thus the magnitude of the reciprocating force. Practical values of the ratio of the connecting-rod length to the crank throw for radial aircraft engines lie in the range from 3.0 to 5.0. It has been found that changes in the ratio of rod length to crank throw (due to a change in the connecting-rod length) within this range affect the resultant crankpin-bearing loads by less than 1 percent.

Stroke. - The effect of a change of stroke at any value of crank angle may be determined from figures 6, 7, and 9 by use of an equivalent speed. The following expression for the equivalent speed due to a change in stroke was derived from equation (5) following the method used for a change of reciprocating weight:

$$N_S = \left(\frac{L_S'}{L_S}\right)^{1/2} N \qquad (12)$$

where

L_S standard stroke, inches

L_S' new stroke, inches

N_S equivalent engine speed with standard stroke, rpm

N actual engine speed with new stroke, rpm

The relative influence of a 10-percent increase or a 10-percent decrease of the mean stroke upon the maximum and mean bearing loads

is illustrated for take-off conditions (2500 rpm and 245 lb/sq in.) in the following table, which was obtained from equation (12) and figures 8 and 10:

Standard stroke L_S		10-percent increase in stroke $L_S' = 1.1 \, L_S$				10-percent decrease in stroke $L_S' = 0.9 \, L_S$			
		W		$\bar{W}$		W		$\bar{W}$	
W (lb)	$\bar{W}$ (lb)	(lb)	(percent)	(lb)	(percent)	(lb)	(percent)	(lb)	(percent)
39,500	33,000	43,500	10.1	36,500	10.6	35,000	-11.4	28,500	-13.6

Bore. - When the principle of similitude (reference 10) is applied to equation (1), it is seen that the effect of a change of bore upon the function Ω may be accomplished by suitable changes of stroke and speed. It will be shown that the crankpin-bearing load corresponding to any bore at any value of crank angle may be determined from figures 6, 7, and 9 by use of an equivalent engine speed and a load correction factor.

The function Ω in equation (1) will change in the same way if, instead of varying the bore, L_S and N are adjusted as indicated in the following formulas:

$$\left(L_S\right)_D = \left(\frac{D}{D'}\right) L_S \tag{13}$$

$$N_D' = \left(\frac{D}{D'}\right)^{1/2} N \tag{14}$$

where

$\left(L_S\right)_D$ equivalent stroke with standard bore and equivalent engine speed, inches

D standard bore, inches

D' new bore, inches

N actual engine speed with new bore, rpm

N_D' equivalent engine speed with standard bore and equivalent stroke, rpm

Because from equation (12) a change in bearing load due to a change in stroke may be determined by use of an equivalent engine speed, equation (13) may be replaced by a second equivalent speed, which when combined with equation (14) yields a single resultant equivalent speed:

$$N_D = \left(\frac{D}{D'}\right) N \tag{15}$$

This equivalent engine speed N_D with standard bore and stroke will change the function Ω in the same way as a change in bore.

Inasmuch as L_S appears outside of the function Ω in equation (1) and has been considered to vary inversely with D in the foregoing discussion, it is evident that a correction factor K, where

$$K = \left(\frac{D'}{D}\right)^2 \tag{16}$$

must be applied to the load obtained from the charts (figs. 6, 8, 9, and 10) by use of the equivalent speed N_D. In the expression for the load angle σ (equation (2)) L_S does not appear outside the function Ω' and thus no load-angle correction factor need be used. The load angle for any bore may be obtained from figure 7 by use of the equivalent engine speed N_D.

The relative influence of a 10-percent increase or decrease of bore upon the maximum and mean bearing loads is illustrated for take-off conditions (2500 rpm and 245 lb/sq in.) in the following table, the values of which were obtained from equations (15) and (16) and figures 8 and 10:

Standard bore D		10-percent increase in bore D' = 1.1 D				10-percent decrease in bore D' = 0.9 D			
		W		$\overline{W}$		W		$\overline{W}$	
W (lb)	$\overline{W}$ (lb)	(lb)	(per-cent)	(lb)	(per-cent)	(lb)	(per-cent)	(lb)	(per-cent)
39,500	33,000	39,000	-1.3	31,500	-4.6	40,000	1.3	34,500	4.6

Compression ratio. - The compression ratio affects the shape of the indicator diagram and therefore the gas force developed in the engine cylinder. The effect of compression ratio upon gas force during the exhaust stroke, the intake stroke, and most of the compression stroke is quite small. The compression ratio has a considerable effect upon the gas force, however, during that portion of the expansion stroke when the piston is near the top-center position.

Inasmuch as the nondimensional compression ratio r enters equations (1) and (2) apart from the other variables, an equivalent speed or indicated mean effective pressure cannot be used to compensate for a change of compression ratio (that is, the principle of similitude cannot be applied). It is therefore necessary to construct a new indicator diagram and repeat the analysis for each different value of compression ratio. It is seen in the following table, however, that the maximum and mean crankpin-bearing loads change very little when the compression ratio changes from 6.0 to 7.4.

Crank angle (deg)	Standard compression ratio		10-percent increase in compression ratio				10-percent decrease in compression ratio			
			W		σ		W		σ	
	W (lb)	σ (deg)	(lb)	(percent decrease)	(deg)	(percent difference)	(lb)	(percent increase)	(deg)	(percent difference)
0	34,000	12.5	32,500	4.4	10.0	-20.0	35,000	2.9	10.5	-16.0
10	22,500	36.0	21,000	6.7	38.5	6.9	25,500	13.3	32.5	-9.7
20	25,500	56.0	24,000	5.9	61.0	8.9	27,000	5.9	51.5	-8.0
30	32,000	58.5	31,500	1.6	59.5	1.7	33,000	3.1	57.5	-1.7
40	37,000	61.0	36,500	1.4	61.0	0	37,500	1.4	61.0	0
50	39,500	65.5	38,500	2.5	65.0	.8	39,500	0	66.0	.8
60	38,500	71.5	38,000	1.3	71.0	-.7	39,500	2.6	71.5	0
70	36,500	79.0	36,000	1.4	79.0	0	38,000	4.1	79.5	.6
Mean	33,000	-----	32,000	3.0	----	-----	34,000	3.0	----	-----

Maximum-load and mean-load charts (figs. 15 and 16) and polar diagrams (fig. 17) with respect to the crank axis are presented for the high values of compression ratios of 8.0 and 9.0 where the loads were significantly altered.

Summary of individual effects of engine dimensions upon crankpin-bearing loads. - The individual effects of the several engine dimensions upon the mean and the maximum crankpin-bearing loads of engine A at take-off conditions are summarized as follows:

Dimension	Change in dimension			
	10-percent increase		10-percent decrease	
	Increase in maximum load (percent)	Increase in mean load (percent)	Increase in maximum load (percent)	Increase in mean load (percent)
Reciprocating weight	6.3	6.1	-6.3	-9.1
Rotating weight	5.1	3.3	-5.1	-7.6
Connecting-rod length[a]	0	0	0	0
Stroke	10.1	10.6	-11.4	-13.6
Bore	-1.3	-4.6	1.3	4.6
Compression ratio	-2.5	-3.0	0	3.0

[a]The change of the mean and maximum bearing load due to a 10-percent increase or decrease in connecting-rod length is too small to warrant a detailed analysis.

APPLICATION OF THE DIMENSIONAL-ANALYSIS METHOD TO OTHER ENGINES

HAVING NINE CYLINDERS PER CRANKPIN

Method. - By use of an equivalent speed and load correction factor, as discussed in the foregoing section, the charts shown in figures 6 to 10 are applicable to any engine having nine cylinders per crankpin. The following equation of over-all equivalent engine speed N_e for changes of reciprocating weight, rotating weight, stroke, and bore was obtained by combining equations (10), (11), (12), and (15):

$$N_\Theta = \frac{D \left[L_S \, L_S{}' \, (F_c + 0.516 \, F_i{}') \, (F_c{}' + 0.516 \, F_i) \right]^{1/2}}{D' \, L_S \, (F_c + 0.516 \, F_i)} \, N \qquad (17)$$

An equivalent speed factor C is obtained by substituting standard values from appendix C into equation (17):

$$C = \frac{N_\Theta}{N} = 0.0315 \frac{\left[L_S{}' \, (32.72 + 0.516 \, F_i{}') \, (F_c{}' + 41.50) \right]^{1/2}}{D'} \qquad (18)$$

A load correction factor for a change of bore is given in
equation (16). This expression reduces to the following equation
when D is replaced by the value for the standard bore given in
appendix C:

$$K = 0.0266 \ (D')^2 \tag{19}$$

The alinement charts of figure 18 are presented in order that C
and K may be conveniently determined for any engine having nine
cylinders per crankpin. The foregoing method is illustrated by
applying it to two additional nine-cylinder radial engines.

Production engine B. - The specifications of engine B are
given in appendix D. From figure 18 the equivalent-speed factor
is seen to be 0.986 and the load correction factor, 0.880. The
crankpin-bearing loads at any engine speed or indicated mean effec-
tive pressure may be determined from figures 6 to 10 by using the
equivalent-speed equation $(N_0 = CN)$ and the load correction
factor K. For example, the maximum and mean load on the crankpin
of engine B at an engine speed of 2600 rpm and an indicated mean
effective pressure of 300 pounds per square inch are found to be
35,800 and 29,100 pounds, respectively, by use of figures 8 and 10.

In order to verify the use of the equivalent speed and the load
correction factors, a polar diagram was constructed for engine B in
the conventional manner at an engine speed of 3000 rpm and an indi-
cated mean effective pressure of 368 pounds per square inch. The
values obtained from the load charts are compared with the values
from the conventional analysis in the following table:

Crank angle (deg)	From conventional analysis		From load charts		Percentage difference	
	W (lb)	σ (deg)	W (lb)	σ (deg)	W	σ
0	40,500	11.5	40,500	11.5	0.0	0.0
10	26,500	39.5	26,000	40.0	1.9	1.3
20	30,500	60.5	31,000	61.0	1.6	.8
30	39,000	61.0	39,500	61.0	1.3	.0
40	45,500	63.5	45,500	63.0	.0	.8
50	48,500	67.0	48,500	67.0	.0	.0
60	47,000	72.5	47,500	72.0	1.1	.7
70	44,000	80.0	44,500	80.0	1.1	.0
Mean	40,000	---------	39,500	---------	1.3	-------

The close agreement between the values obtained is considered an adequate check of the two methods used herein. Speed and load scales for this engine have been included on the polar diagrams given in figures 12 and 17.

Production engine C. -- The specifications of engine C are given in appendix E. From figure 18 the equivalent speed factor is seen to be 0.955 and the load correction factor, 1.000. The maximum and mean crankpin-bearing loads for this engine at an engine speed of 2600 rpm and an indicated mean effective pressure of 300 pounds per square inch are found to be 37,700 and 30,300 pounds by use of figures 8 and 10. Speed and load scales for this engine have been included on figures 12 and 17.

APPLICATION OF THE DIMENSIONAL-ANALYSIS METHOD TO A DESIGN PROBLEM

The foregoing analysis enables a broad evaluation of the relative influence of both operating and design variables. Several examples have been presented to illustrate the use of this analysis in predicting the change in crankpin-bearing load that is brought about by a change of the operating variables. Although the several design variables have been independently treated, these quantities are interrelated and one cannot be changed without influencing the others. The following discussion illustrates the manner in which the foregoing analysis may be applied to a design problem.

It is customary in engineering design to evaluate the performance of a device in terms of a relative efficiency. A group of possible engines, having nine cylinders per crankpin, might be considered on the basis of the relative magnitude of the crankpin-bearing load per indicated horsepower developed. Many other design criteria must, of course, be considered in arriving at the most desirable engine dimensions.

In order to determine approximately the variation of bearing load per unit indicated horsepower with the bore of a radial engine having a fixed stroke, the reciprocating and the rotating weights are assumed to vary as the cube of the bore, according to geometric similarity. The reciprocating and the rotating weights per crankpin may then be expressed

$$F_i' = K_1 (D')^3 \qquad\qquad (20)$$

$$F_c' = K_2 (D')^3 \qquad\qquad (21)$$

The corresponding values of K_1 and K_2 for the three engines are as follows:

Engine	K_1	K_2
A	0.349	0.142
B	.390	.182
C	.335	.148

The equivalent speed factor C is obtained by substituting equations (20) and (21) into equation (18):

$$C = \frac{N_e}{N} \frac{0.0315\left\{L_S'\left[32.72 + 0.516\,K_1\,(D')^3\right]\left[K_2\,(D')^3 + 41.5\right]\right\}^{1/2}}{D'}$$

$$(22)$$

It is evident from figure 8 and equation (5) that the maximum bearing load for engine A in the practicable operating region is given by the following expression to a good approximation:

$$W = 7235 \times 10^{-6}\,N^2 \tag{23}$$

For any other engine this equation becomes

$$W = 7235 \times 10^{-6}\,K\,N_e^2 \tag{24}$$

where K is the load correction factor given in equation (19) and N_e is the equivalent engine speed from equation (22).

The indicated horsepower per crankpin developed by any engine is:

$$ihp = 8.91 \times 10^{-6}\,L_S'\,(D')^2\,p\,N \tag{25}$$

from equations (19), (22), (24), and (25)

$$\frac{W}{ihp} = \left[0.0111\,K_1K_2(D')^4 + (0.4588\,K_1 + 0.7012\,K_2)(D') + 29.10(D')^{-2}\right]\frac{N}{p}$$

$$(26)$$

The quantity in brackets in equation (26) (the specific bearing-load factor) is plotted against bore in figure 19 for engines A, B, and C. These curves show the variation of specific bearing load with bore for a given stroke. It is evident that the minimum value of specific bearing load is obtained for a bore-stroke ratio slightly less than 1. The dimensions of the three engines considered are such that the respective values of specific bearing load lie close to the optimum values.

DISCUSSION

The maximum and mean loads acting on the crankpin bearing of a radial engine increase significantly with speed in the practicable operating region. An increase in indicated mean effective pressure causes a slight reduction in the bearing loads existing at practicable values of engine speed. Optimum combinations of indicated mean effective pressure and engine speed for which the maximum bearing load is a minimum at any power level are seen to exist (fig. 8), but such optimum conditions do not lie in the range of practicable engine operation. When a radial engine is operated at full-throttle setting the maximum and mean crankpin-bearing loads will be minima for the engine speed obtaining. (See figs. 8 and 10.)

It can be seen in figure 12 that the difference between the maximum and minimum loads increases directly with indicated mean effective pressure. Figure 17 shows that this range of load is accentuated at high values of compression ratio. If the range of stress to which a bearing is subjected is taken as the criterion of fatigue severity, bearings operating under high values of indicated mean effective pressure or in conjunction with high values of compression ratio should be the first to fail by fatigue.

Representative values of the crankpin-bearing operating characteristics of production engines A, B, and C, obtained from the maximum and mean load charts of this report, are given in table I.

CONCLUSION

A method of computing the load acting on the crankpin of a radial engine under all operating conditions has been developed by use of dimensional analysis. By the principle of similitude the results obtained for a particular engine are readily applicable to any radial engine having the same number of cylinders.

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APPENDIX A

NOTATION

C equivalent-speed factor

D diameter of bore, in.

D' new bore, in.

F_C rotating weight per crankpin, lb

F_C' new rotating weight per crankpin, lb

F_i reciprocating weight per crankpin, lb

F_i' new reciprocating weight per crankpin, lb

F_i'' reciprocating weight per cylinder, lb

K load correction factor

K_1, K_2 constants

L_R length of master connecting rod, in.

L_S stroke, in.

L_S' new stroke, in.

$(L_S)_D$ equivalent stroke with standard bore and equivalent engine speed, in.

M_C rotating mass per crankpin, slugs

M_i reciprocating mass per crankpin, slugs

n number of equally spaced cylinders

N engine speed, rpm

N_C equivalent engine speed with standard rotating weight, rpm

N_D equivalent engine speed with standard bore and stroke, rpm

N_D' equivalent engine speed with standard bore and equivalent stroke, rpm

N_e over-all equivalent engine speed, rpm

N_i equivalent engine speed with standard reciprocating weight, rpm

N_S equivalent engine speed with standard stroke, rpm

p indicated mean effective pressure, lb/sq in.

p_m manifold pressure, lb/sq in. absolute

RF rubbing factor, (ft-lb)/(sq in.)(sec)

r compression ratio

W crankpin-bearing load, lb

$\bar{W}$ mean crankpin-bearing load, lb

W_c rotating inertia force per crankpin, lb

W_i rotating inertia force per crankpin, lb

W_i'' reciprocating inertia force per cylinder, lb

W_r resultant inertia force due to all rotating and reciprocating mass that acts upon crankpin, lb

β, α, ϕ angles defined in figure 1

θ crank angle, deg

σ angle between axis of cylinder 1 and resultant bearing load in the direction of rotation, deg

$\Omega, \Omega', \Omega'', \Omega'''$ functions

Subscript

m individual cylinder number

APPENDIX B

THE TOTAL INERTIA FORCE ACTING UPON THE CRANKPIN OF A RADIAL ENGINE

The analyses discussed in the body of the paper utilize an expression for the resultant inertia force acting upon the crankpin. This total inertia force is the vector sum of the rotating inertia force and the individual reciprocating inertia forces for each cylinder. The rotating inertia force per crankpin W_C may be determined from the following equation:

$$W_C = 28.4 \times 10^{-6} \ F_c \ \frac{L_S}{2} \ N^2 \tag{B1}$$

The following expression for the reciprocating inertia force per cylinder W_i'' is given in reference 11 (with a change in notation) to a very good approximation:

$$W_i'' = 28.4 \times 10^{-6} \ F_i'' \ \frac{L_S}{2} \ N^2 \left(\cos \theta + \frac{L_S}{2 \ L_R} \cos 2 \ \theta \right) \tag{B2}$$

where F_i'' is the reciprocating weight per cylinder in pounds

Prescott and Poole (reference 3) give the following equation (with a change of notation) for the resultant inertia force per crankpin:

$$W_r = 28.4 \times 10^{-6} \ \frac{L_S}{2} \ N^2 \left\{ F_c + \left[\frac{n}{2} + \frac{n}{4} \left(\frac{L_S}{2 \ L_R} \right)^2 \right] F_i'' \right\} \tag{B3}$$

This force is reported to act along the crank in the direction of the rotating force W_c. Therefore, from equation (B3) the expression for the reciprocating inertia force W_i per crankpin is

$$W_i = 28.4 \times 10^{-6} \ \frac{L_S}{2} \ N^2 \left[\frac{n}{2} + \frac{n}{4} \left(\frac{L_S}{2 \ L_R} \right)^2 \right] F_i'' \tag{B4}$$

In order that equation (B3) be valid, it is necessary that the vector sum of the n acceleration factors $\left[\cos \theta_m + (L_S/2L_R) \cos 2 \ \theta_m \right]$ be equal in magnitude and direction to the expression $\left[\frac{n}{2} + \frac{n}{4} \left(\frac{L_S}{2 \ L_R} \right)^2 \right]$

(where the subscript m is the number of an individual cylinder of
an engine having nine cylinders). An outline of operations required
to compute analytically the magnitude R and the direction γ of
the vector sum of these n acceleration factors follows:

$$R = \left(X^2 + Y^2\right)^{1/2} \tag{B5}$$

$$\gamma = \tan^{-1} (X/y) \tag{B6}$$

$$X = \sum_{m=0}^{n} \left(\cos \theta_m + \frac{L_S}{2\,L_R} \cos 2\,\theta_m\right) \sec \phi_m \sin \beta_m \tag{B7}$$

$$Y = \sum_{m=0}^{n} \left(\cos \theta_m + \frac{L_S}{2\,L_R} \cos 2\,\theta_m\right) \sec \phi_m \cos \beta_m \tag{B8}$$

$$\phi_m = \sin^{-1}\left(\frac{L_S}{2\,L_R} \sin \theta_m\right) \tag{B9}$$

and where

m	θ_m	β_m
1	θ	$n\left(\frac{360}{n}\right) - \phi_m$
$n-1$	$\frac{720}{n} + \theta$	$(n-2)\left(\frac{360}{n}\right) - \phi_m$
$n-3$	$2\left(\frac{720}{n}\right) + \theta$	$(n-4)\left(\frac{360}{n}\right) - \phi_m$
$n-5$	$3\left(\frac{720}{n}\right) + \theta$	$(n-6)\left(\frac{360}{n}\right) - \phi_m$
. . .	. . .	. . .
2	$\left(\frac{n-1}{2}\right)\left(\frac{720}{n}\right) + \theta$	$\left(\frac{360}{n}\right) - \phi_m$
n	$\left(\frac{n+1}{2}\right)\left(\frac{720}{n}\right) + \theta$	$(n-1)\left(\frac{360}{n}\right) - \phi_m$
$n-2$	$\left(\frac{n+3}{2}\right)\left(\frac{720}{n}\right) + \theta$	$(n-3)\left(\frac{360}{n}\right) - \phi_m$
$n-4$	$\left(\frac{n+5}{2}\right)\left(\frac{720}{n}\right) + \theta$	$(n-5)\left(\frac{360}{n}\right) - \phi_m$
. . .	. . .	. . .
3	$(n-1)\left(\frac{720}{n}\right) + \theta$	$2\left(\frac{360}{n}\right) - \phi_m$

and where β_m is the angle between the Y axis (fig. 1) and the
connecting rod of cylinder m.

The vector sum of the n acceleration factors given by Prescott and Poole (reference 3) is compared with the exact analytical procedure for a number of representative cases in table II. It is evident that the resultant inertia force may be obtained to a very good approximation from equation (B3) for radial engines having 5, 7, 9, or 11 cylinders. The equation does not apply to a three-cylinder engine.

APPENDIX C

SPECIFICATIONS OF PRODUCTION ENGINE A

Number of cylinders 9
Arrangement of cylinders . . . . single-row radial
Numbering of cylinders viewing
   antipropeller end . . . . . . clockwise; top cylinder, number 1
Firing order 1, 3, 5, 7, 9, 2, 4, 6, 8
Direction of crankshaft rotation
   viewing antipropeller end . . . . clockwise
Bore, in. 6.125
Stroke, in. 6.875
Piston area, sq in. 29.46
Engine speed at take-off, rpm 2500
Indicated mean effective pressure
   at take-off, lb/sq in. 245
Brake mean effective pressure at
   take-off, lb/sq in. 208
Assumed mechanical efficiency at
   take-off, percent 85
Manifold pressure at take-off,
   in. Hg absolute 47.5
Compression ratio 6.70
Master-rod length, in. 13.75
Articulated-rod length, in. 10.81
Ratio of master connecting-rod
   length to crank throw 4.00
Spark advance (both plugs), deg B.T.C. 20
Valve timing:
   Intake valve opens, deg A.T.C. 15
   Intake valve closes, deg A.B.C. 44
   Exhaust valve opens, deg B.B.C. 74
   Exhaust valve closes, deg B.T.C. 25
Crankpin diameter, in. 3.250
Effective length of crankpin bearing, in. 3.11
Projected crankpin bearing area, sq in. 10.10
Reciprocating and rotating weights:
   Total weight of piston assembly, lb 7.13
   Weight of upper end of master rod, lb 3.36
   Weight of upper end of articulated rod, lb 1.60
   Average total reciprocating weight per cylinder, lb 8.93
   Weight of lower end of master rod, lb 13.62
   Weight of crankpin bearing, lb 1.19
   Weight of lower end of articulated rod, lb 1.39
   Weight of knuckle pin, lb 0.675
   Total weight of small rotating parts, lb 1.39
   Total rotating weight per crankpin, lb 32.72

APPENDIX D

SPECIFICATIONS OF PRODUCTION ENGINE B

Number of cylinders 18
Arrangement of cylinders . . . . double-row radial
Numbering of cylinders viewing
   antipropeller end . . . . . clockwise; top cylinder of rear row,
                    number 1; odd numbers in rear row
Firing order 1, 12, 5, 16, 9, 2, 13, 6, 17,
                       10, 3, 14, 7, 18, 11, 4, 15, 8
Direction of crankshaft rotation
   viewing antipropeller end . . . . clockwise
Bore, in. 5.750
Stroke, in. 6.000
Piston area, sq in. 25.97
Engine speed at take-off, rpm 2800
Indicated mean effective pressure
   at take-off, lb/sq in. 249
Brake mean effective pressure
   at take-off, lb/sq in. 212
Assumed mechanical efficiency at
   take-off, percent 85
Manifold pressure at take-off,
   in. Hg absolute 55
Compression ratio 6.75
Master-rod length, in. 12.312
Articulated-rod length, in. 9.271
Ratio of master connecting-rod
   length to crank throw 4.104
Spark advance (both plugs), deg B.T.C. 20
Valve timing:
   Intake valve opens, deg A.T.C. 36
   Intake valve closes, deg A.B.C. 60
   Exhaust valve opens, deg B.B.C. 70
   Exhaust valve closes, deg B.T.C. 26
Crankpin diameter, in. 3.500
Effective length of crankpin bearing, in. 3.06
Projected crankpin bearing area, sq in. 10.71
Reciprocating and rotating weights:
   Total weight of piston assembly, lb 6.70
   Weight of upper end of master rod, lb 2.71
   Weight of upper end of articulated rod, lb 1.39
   Average total reciprocating weight per cylinder, lb 8.24
   Weight of lower end of articulated rod, lb 1.30
   Total rotating weight per crankpin, lb 34.57

APPENDIX E

SPECIFICATIONS OF PRODUCTION ENGINE C

Number of cylinders 18
Arrangement of cylinders . . . . double-row radial
Numbering of cylinders viewing
    antipropeller end . . . . . clockwise; top cylinder of rear row,
                        number 1; odd numbers in rear row
Firing order 1, 12, 5, 16, 9, 2, 13, 6, 17,
                      10, 3, 14, 7, 18, 11, 4, 15, 8
Direction of crankshaft rotation
    viewing antipropeller end . . . . clockwise
Bore, in. 6.125
Stroke, in. 6.312
Piston area, sq in. 29.47
Engine speed at take-off, rpm 2800
Indicated mean effective pressure
    at take-off, lb/sq in. 219
Brake mean effective pressure
    at take-off, lb/sq in. 186
Assumed mechanical efficiency at
    take-off, percent 85
Manifold pressure at take-off,
    in. Hg absolute 46
Compression ratio 6.85
Master-rod length, in. 13.937
Articulated-rod length, in. 10.81
Ratio of master connecting-rod
    length to crank throw 4.42
Spark advance (both plugs), deg B.T.C. 20
Valve timing:
    Intake valve opens, deg A.T.C. 15
    Intake valve closes, deg A.B.C. 44
    Exhaust valve opens, deg B.B.C. 74
    Exhaust valve closes, deg B.T.C. 25
Crankpin diameter, in. 3.625
Effective length of crankpin bearing, in. 3.03
Projected crankpin bearing area, sq in. 10.98
Reciprocating and rotating weights:
    Total weight of piston assembly, lb 6.77
    Weight of upper end of master rod, lb 3.33
    Weight of upper end of articulated rod, lb 1.60
    Average total reciprocating weight per cylinder, lb 8.57
    Weight of lower end of master rod, lb 14.24
    Weight of crankpin-bearing, lb 1.53
    Weight of lower end of articulated rod, lb 1.39
    Total weight of knuckle pin, lb 0.66
    Weight of small rotating parts, lb 1.79
    Total rotating weight per crankpin, lb 33.96

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TABLE I - CRANKPIN-BEARING OPERATING CHARACTERISTICS FOR

PRODUCTION ENGINES A, B, AND C

Engine	Power condi- tion	Engine speed (rpm)	Indi- cated mean effec- tive pres- sure (lb/sq in.)	Indi- cated horse- power	Maxi- mum bear- ing load (lb)	Maxi- mum unit bear- ing load[a] (lb/ sq in.)	Mean bear- ing load (lb)	Mean unit bear- ing load[a] (lb/ sq in.)	Rubbing factor[b] (ft-lb/ (sq in.) (sec))
A	1	2500	0	0	45,200	4520	45,200	4520	159,000
	2	2500	150	860	41,300	4140	36,500	3650	128,000
	3	2500	250	1440	39,200	3930	32,600	3260	115.000
	4	2500	350	2020	37,800	3770	30,000	3000	105,500
	5	2750	250	1580	48,200	4820	40,900	4090	159,000
	6	3000	250	1720	58,800	5880	51,700	5170	218,000
B	1	2500	0	0	38,500	3590	38,500	3590	138,000
	2	2500	150	1330	35,200	3280	31,000	2890	101,000
	3	2500	250	2220	33,700	3140	27,600	2580	99,000
	4	2500	350	3100	32,100	2990	25,500	2380	91,400
	5	2750	250	2430	41,100	3840	34,600	3220	135,000
	6	3000	250	2660	50,400	4700	43,800	4090	171,000
C	1	2500	0	0	41,200	3760	41,200	3760	149,000
	2	2500	150	1580	37,700	3440	33,000	3010	119,000
	3	2500	250	2640	35,700	3250	29,100	2650	105,000
	4	2500	350	3700	34,400	3140	27,300	2480	98,500
	5	2750	250	2900	43,800	3990	36,400	3310	144,000
	6	3000	250	3170	52,700	4800	46,000	4200	199,000

[a]The effective projected bearing areas are:

Engine	Bearing area (sq in.)
A	10.10
B	10.71
C	10.98

[b]The rubbing factor (reference 3) is computed from the equation
$RF = ANW$ where

Engine	A
A	1.41×10^{-3}
B	1.43
C	1.44

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TABLE II - COMPARISON OF TWO METHODS OF COMPUTING ACCELERATION

FACTORS FOR RESULTANT INERTIA FORCES

$\dfrac{L_S}{2 L_R}$	Number of cylinders n	Crank angle θ (deg)	Vector $\displaystyle\sum_{n=0}^{n}\left(\cos\theta_n + \dfrac{L_S}{2 L_R}\cos 2\theta_n\right)$		$\left[\dfrac{n}{2} + \dfrac{n}{4}\left(\dfrac{L_S}{2 L_R}\right)^2\right]$		Percentage difference	
			Magnitude R	Direction γ (deg)	Magnitude R	Direction γ (deg)	Magnitude	Direction
0.333	3	0	2.348	0.00	1.583	0.00	32.6	0.0
.333	3	60	.826	60.00	1.583	60.00	91.6	.0
0.333	5	0	2.641	0.00	2.639	0.00	0.1	0.0
.333	5	20	2.646	20.01	2.639	20.00	.2	.1
.333	5	80	2.642	80.09	2.639	80.00	.1	.1
.333	5	135	2.643	135.00	2.639	135.00	.1	.0
0.333	7	0	3.720	0.00	3.694	0.00	0.7	0.0
.333	7	20	3.715	19.85	3.694	20.00	.6	.8
.333	7	60	3.696	59.93	3.694	60.00	.1	.1
.333	7	80	3.697	80.12	3.694	80.00	.1	.2
.333	7	100	3.707	100.21	3.694	100.00	.4	.2
0.333	9	0	4.765	0.00	4.750	0.00	0.3	0.0
.333	9	20	4.765	20.00	4.750	20.00	.3	.0
.333	9	40	4.765	40.00	4.750	40.00	.3	.0
.333	9	60	4.789	59.40	4.750	60.00	.8	1.0
0.333	11	0	5.813	0.00	5.806	0.00	0.1	0.0
.333	11	20	5.821	20.10	5.806	20.00	.3	.5
.333	11	60	5.839	59.95	5.806	60.00	.6	.1
0.250	9	0	4.645	0.00	4.641	0.00	0.1	0.0
.250	9	60	4.645	60.00	4.641	60.00	.1	.0
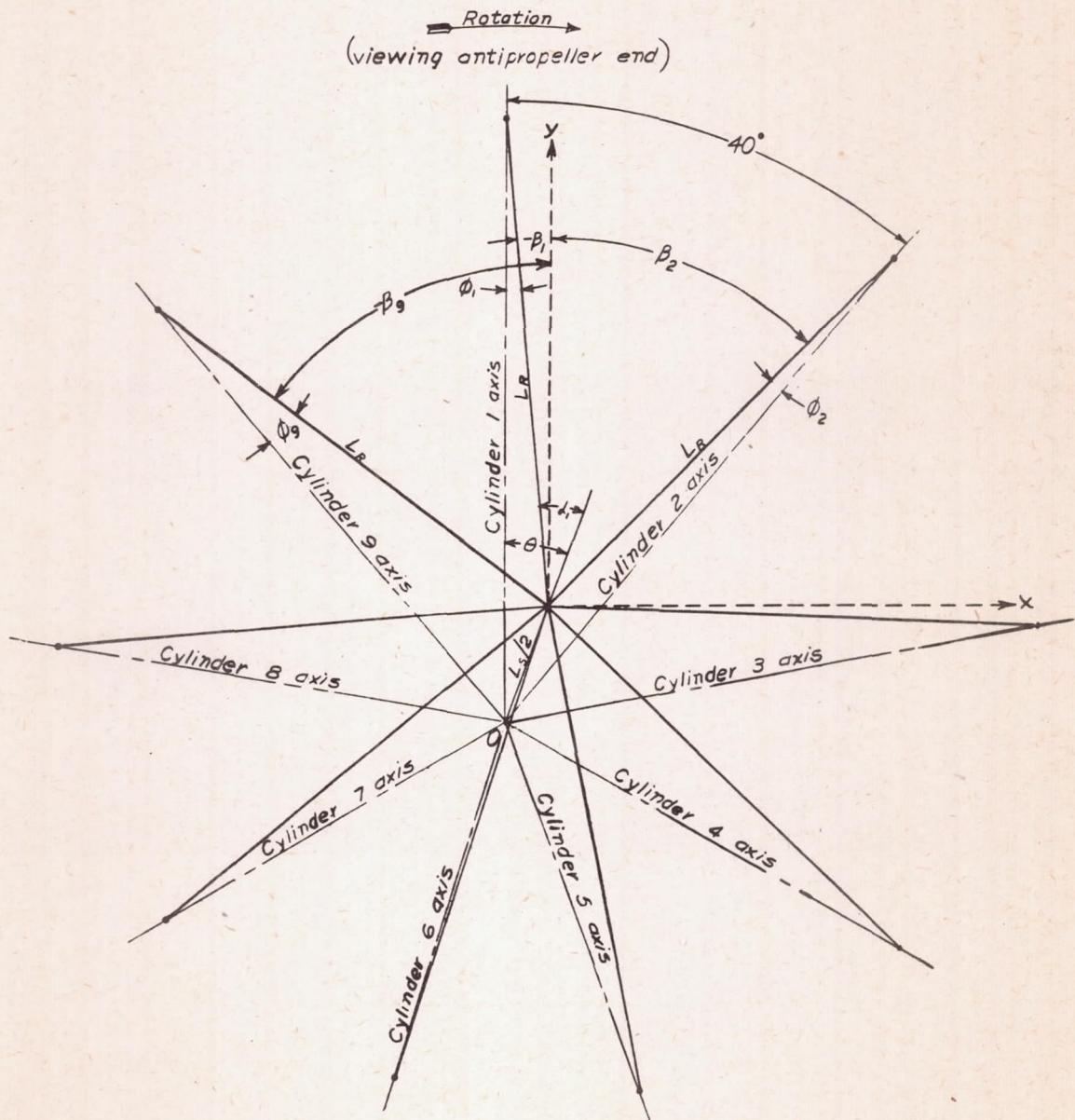
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Figure 1. - Schematic diagram of the mechanism of a nine-cylinder radial engine.

Piston-pin
bushing

Piston-pin bushing

Articulated
rod

Master rod

Locking-plate
screw

Knuckle-pin
lock

Knuckle pin

Master-rod
spacer

End-seal
disk

Knuckle-
pin bushing

End-seal-
disk spring

Knuckle-pin
locking plate

Master-rod
bearing
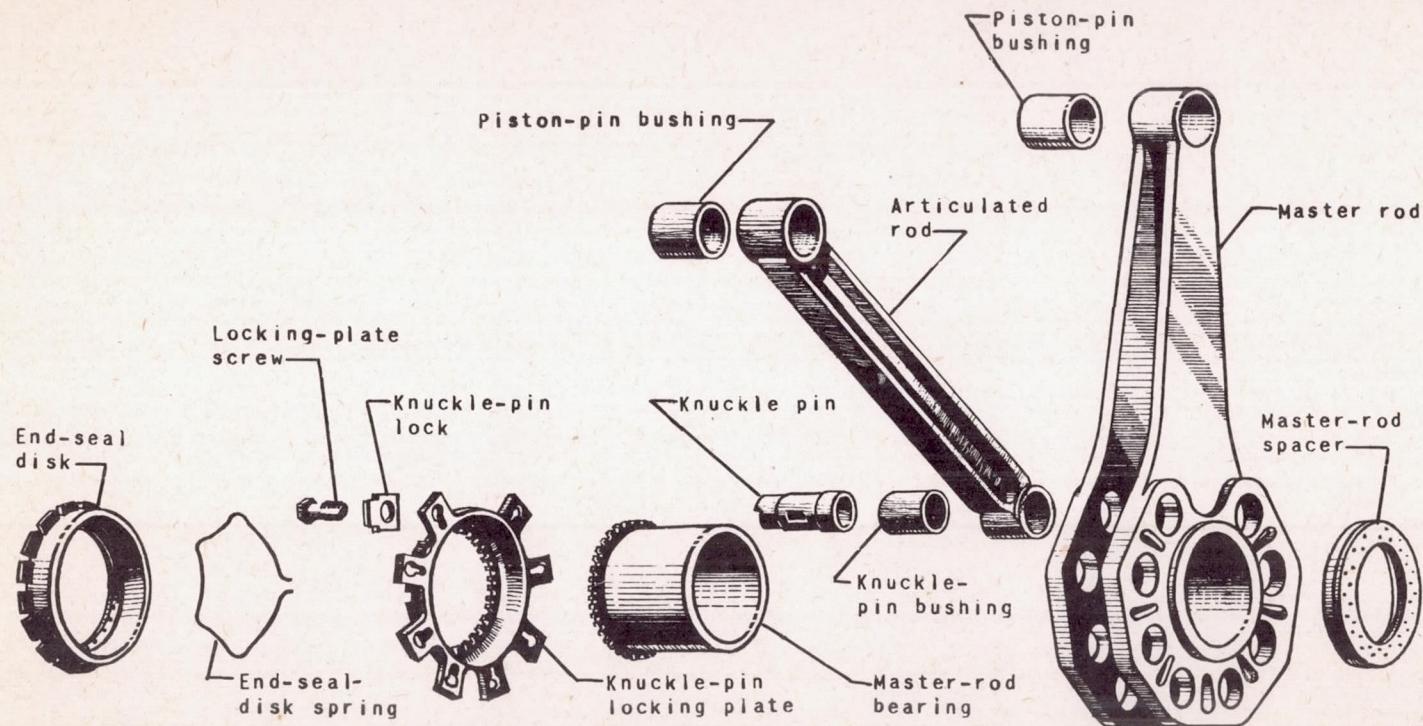
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Figure 2. - Connecting-rod arrangement for engine A.

Fig. 2

Oil-metering screw

Crankpin oil holes

Lock

Rear crankcheek

Front main roller
bearing

Rear main roller
bearing

Crankshaft front section
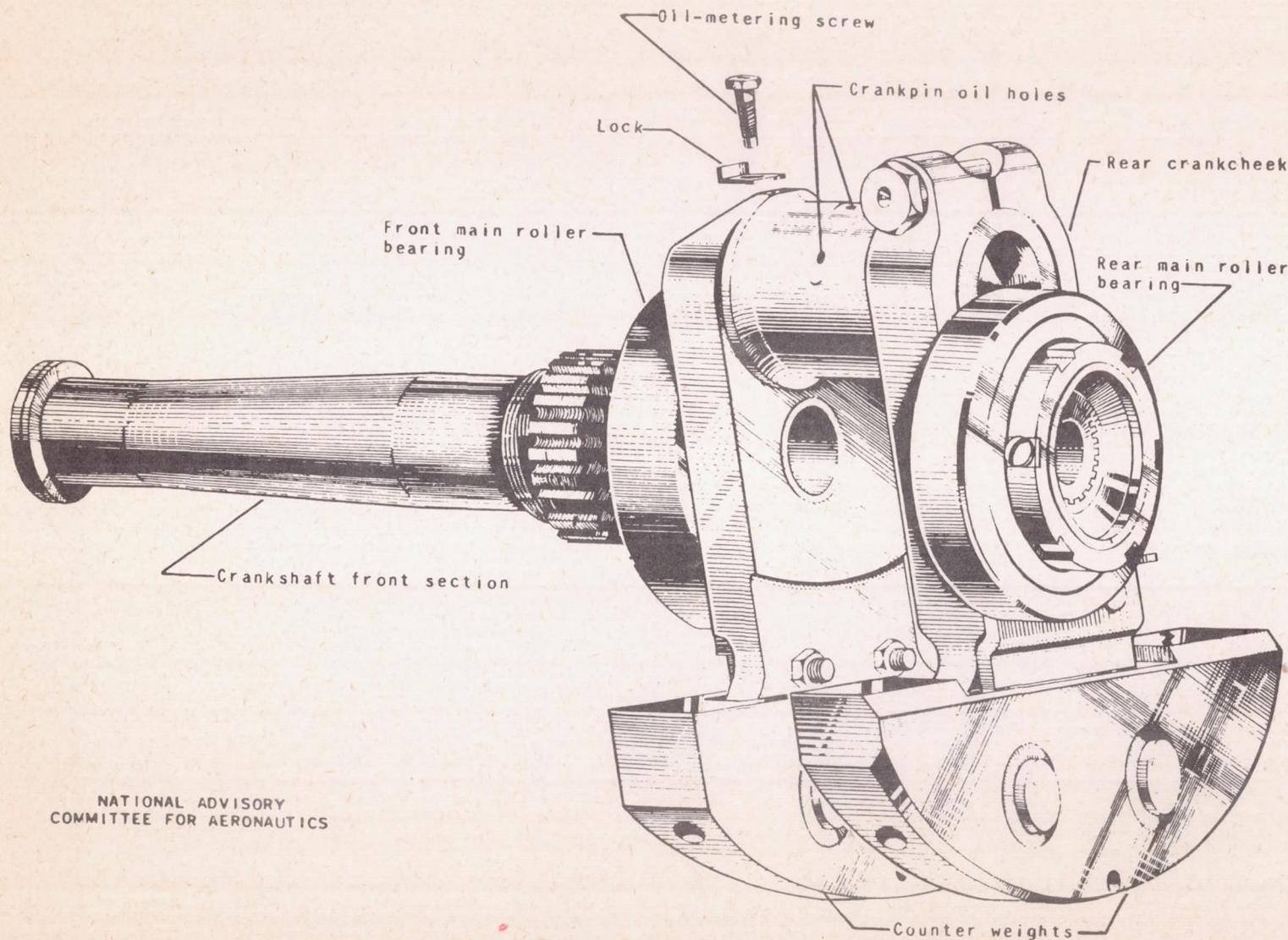
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Counter weights

Figure 3. - Crankshaft arrangement for engine A.

Fig. 3
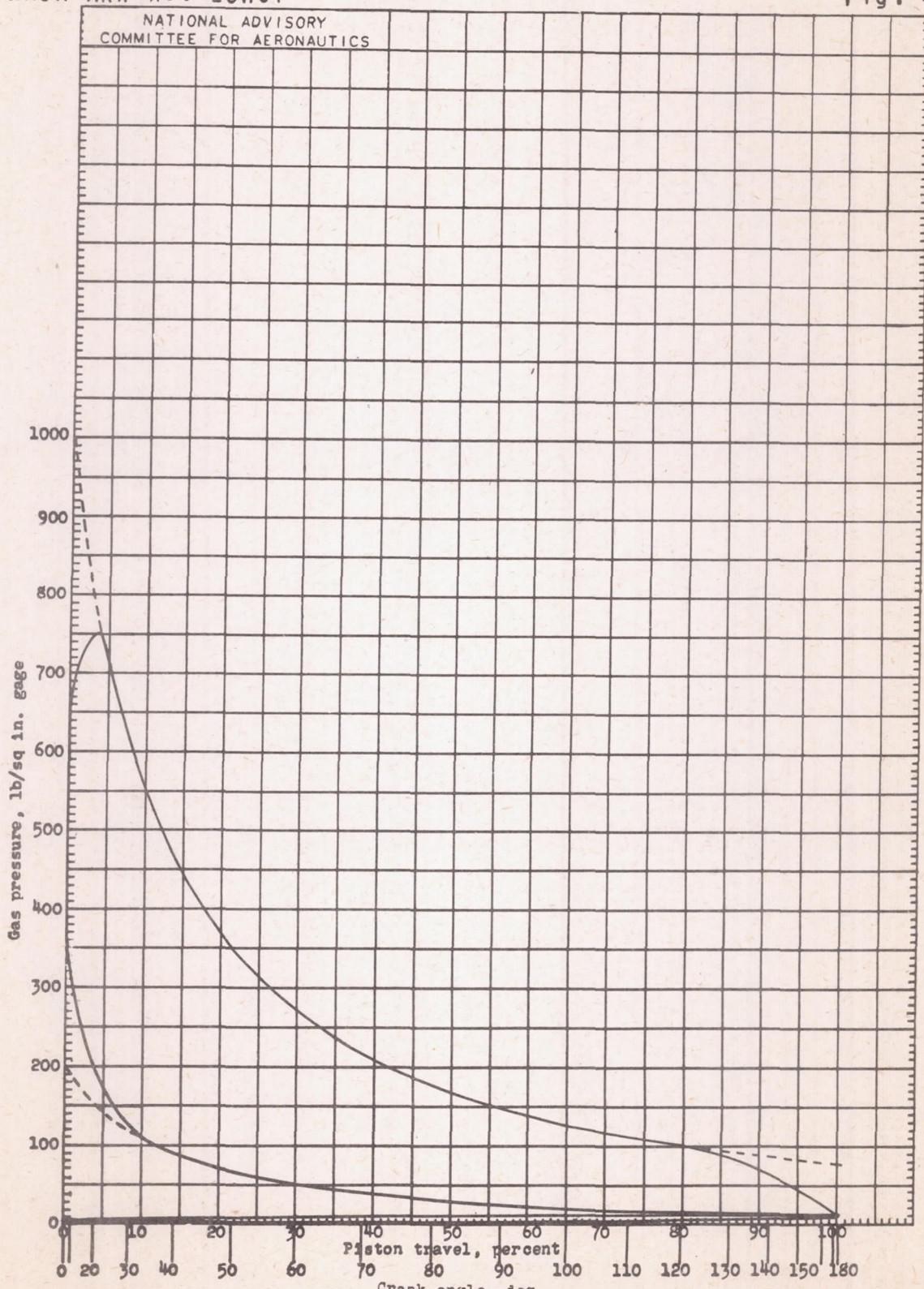
Figure 4. – Theoretical indicator diagram for engine A. Indicated mean effective
pressure, 245 pounds per square inch; engine speed, 2500 rpm; indicator-diagram
factor, 0.90; compression ratio, 6.7; exponents of expansion and compression
curves, 1.30; maximum pressure, 75 percent of computed maximum.
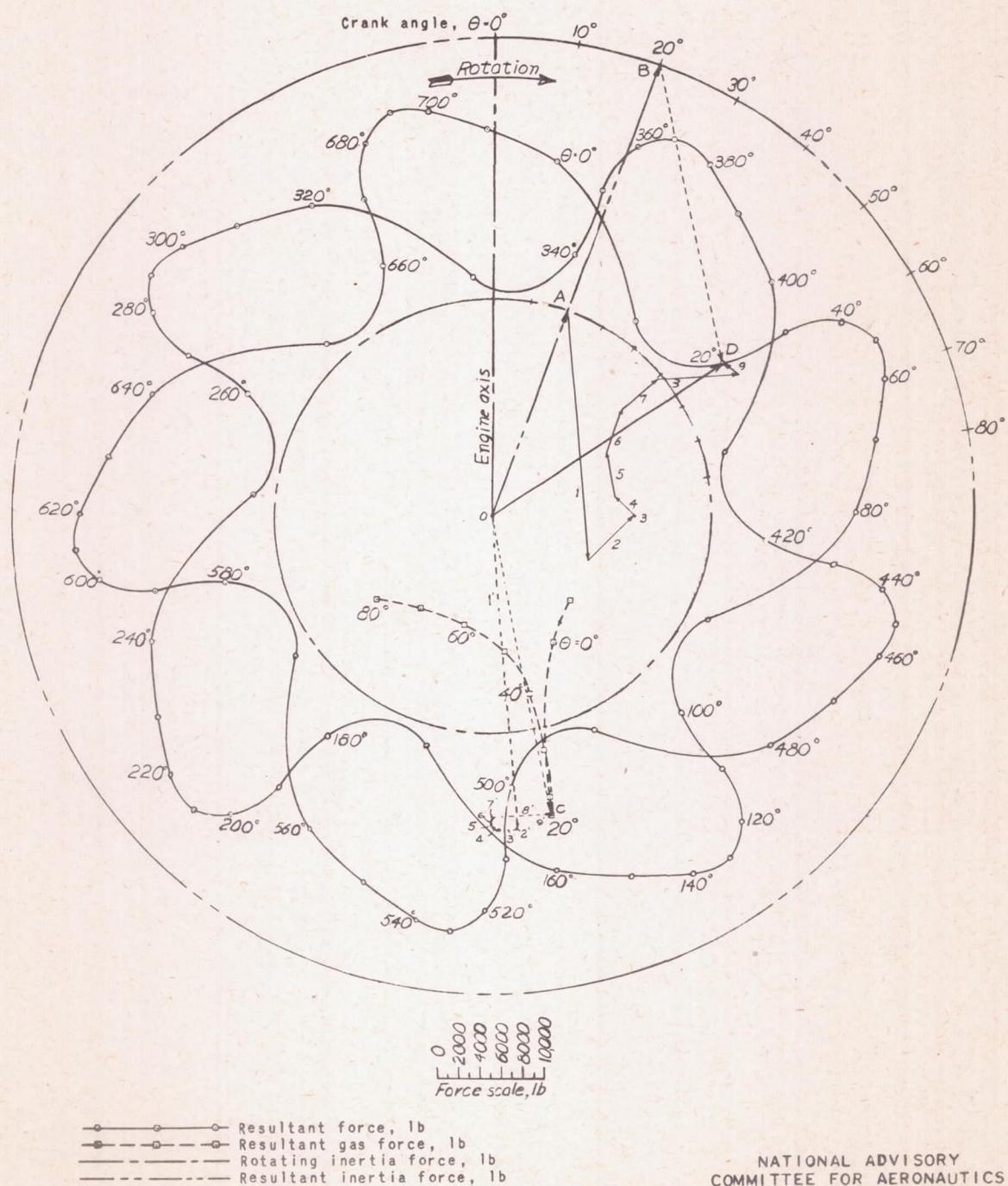
Figure 5. - Polar diagram showing the magnitude of the resultant force on the crankpin of production engine A and its direction with respect to the engine axis. Engine speed, 2500 rpm; indicated mean effective pressure, 245 pounds per square inch; compression ratio, 6.7.
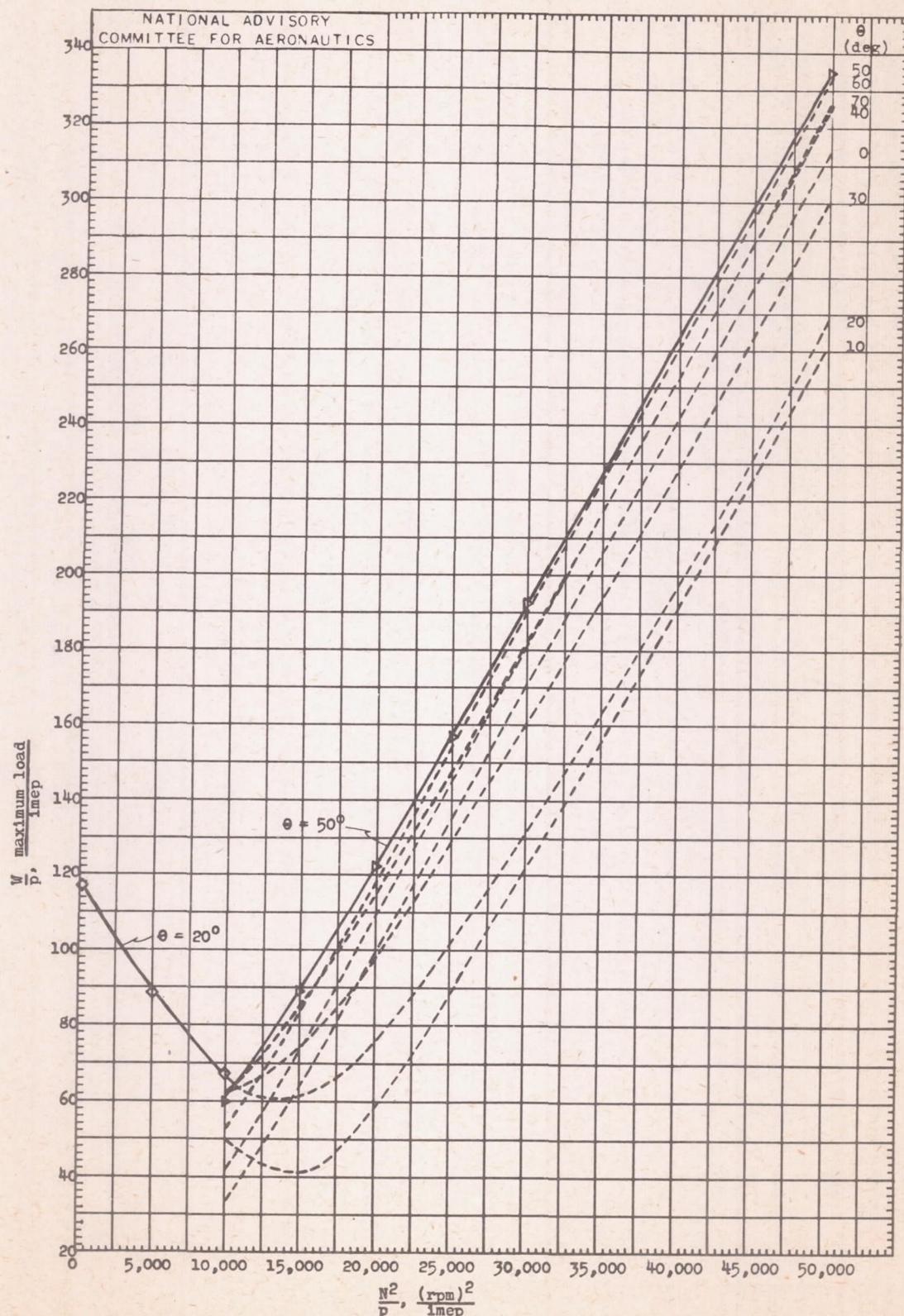
Figure 6. - Variation of W/p with N^2/p for engine A at a compression ratio of 6.7.
(For application of this chart to other radial engines having nine cylinders per crankpin see APPLICATION OF THE DIMENSIONAL ANALYSIS METHOD TO OTHER ENGINES HAVING NINE CYLINDERS PER CRANKPIN.)

(a) θ = 0°, 10°, 20°, 30°.
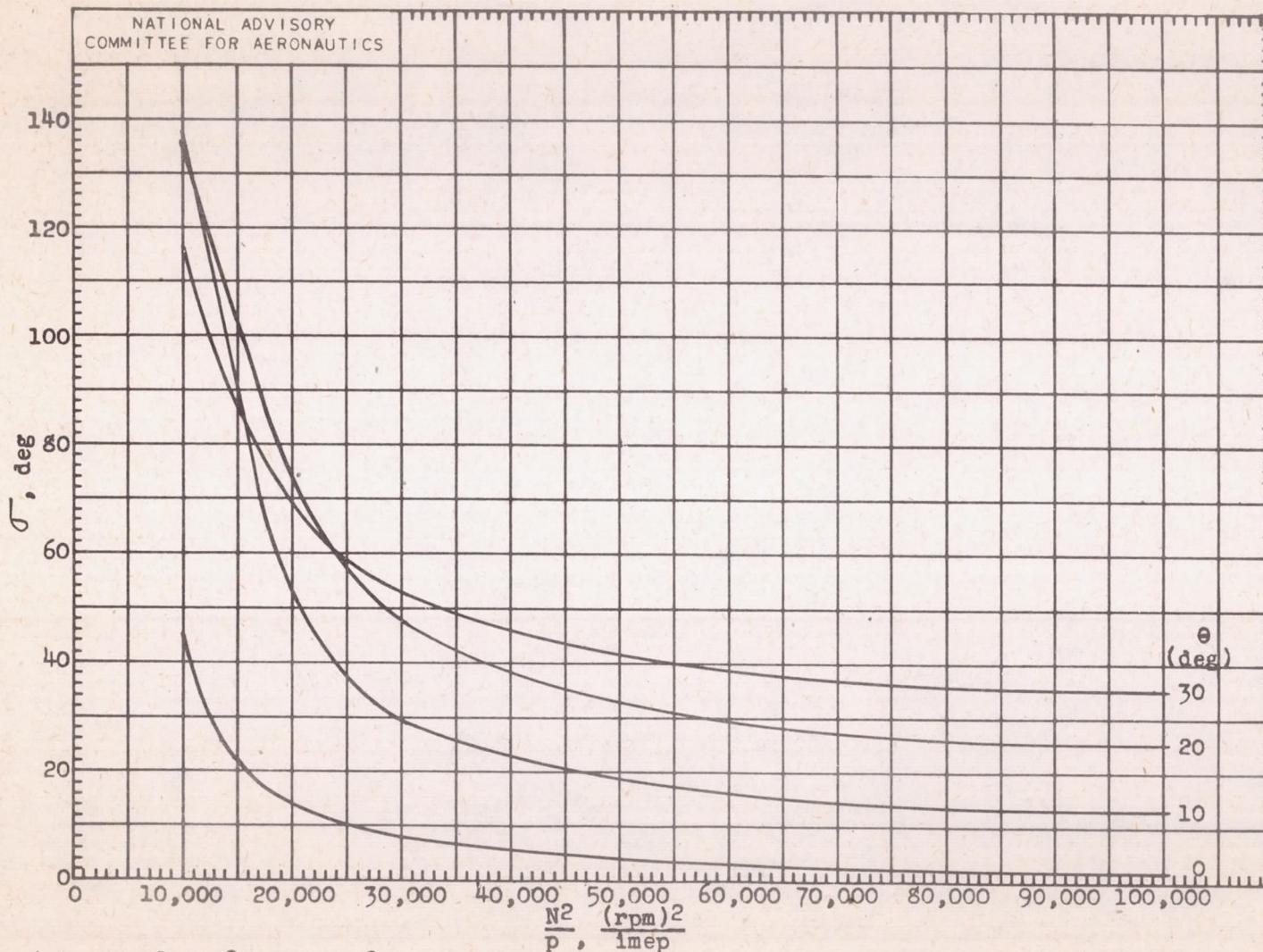
Figure 7. - Variation of σ with N^2/p for engine A at a compression ratio of 6.7.
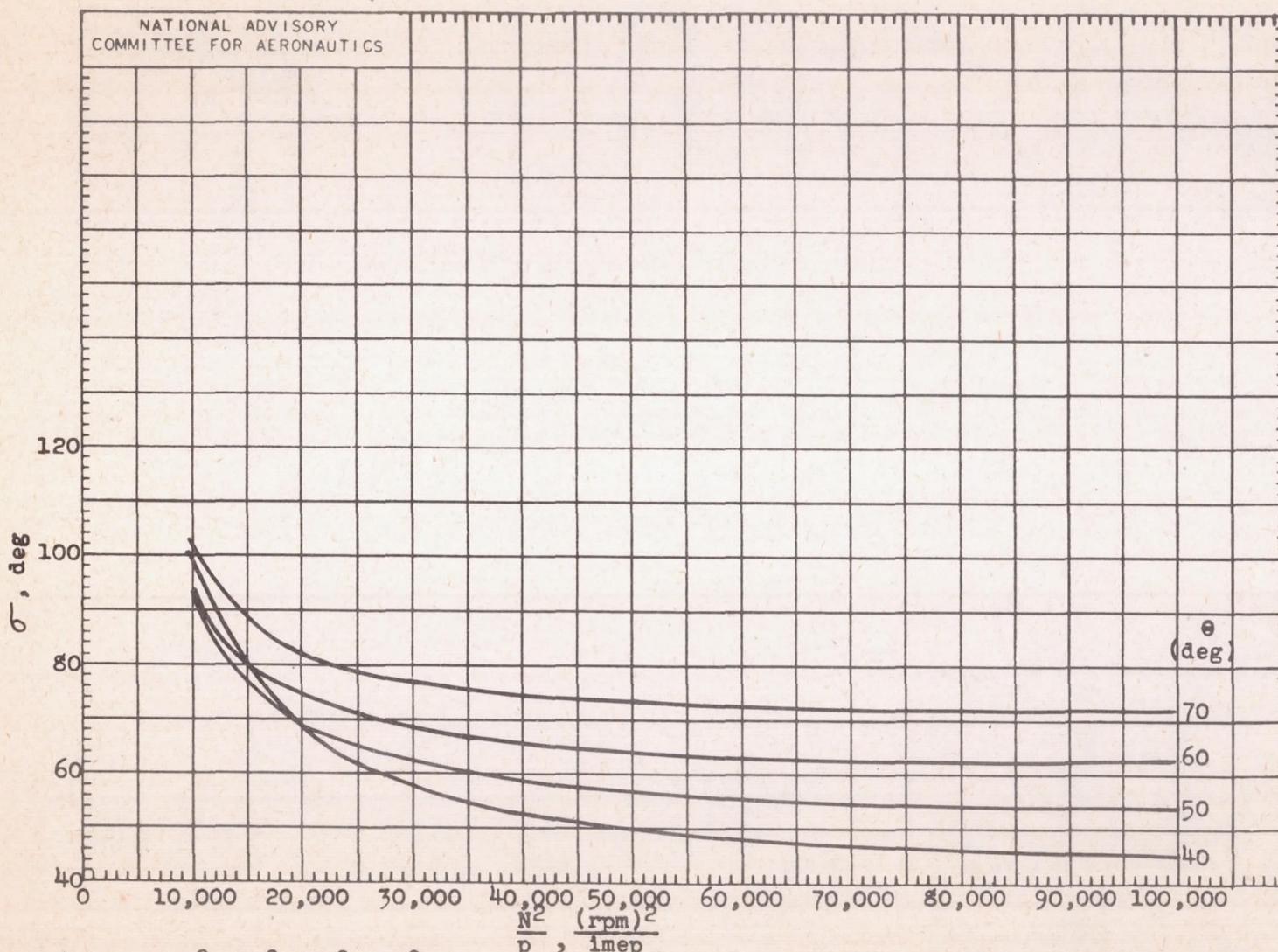
(b) θ = 40°, 50°, 60°, 70°.
Figure 7. – Concluded.
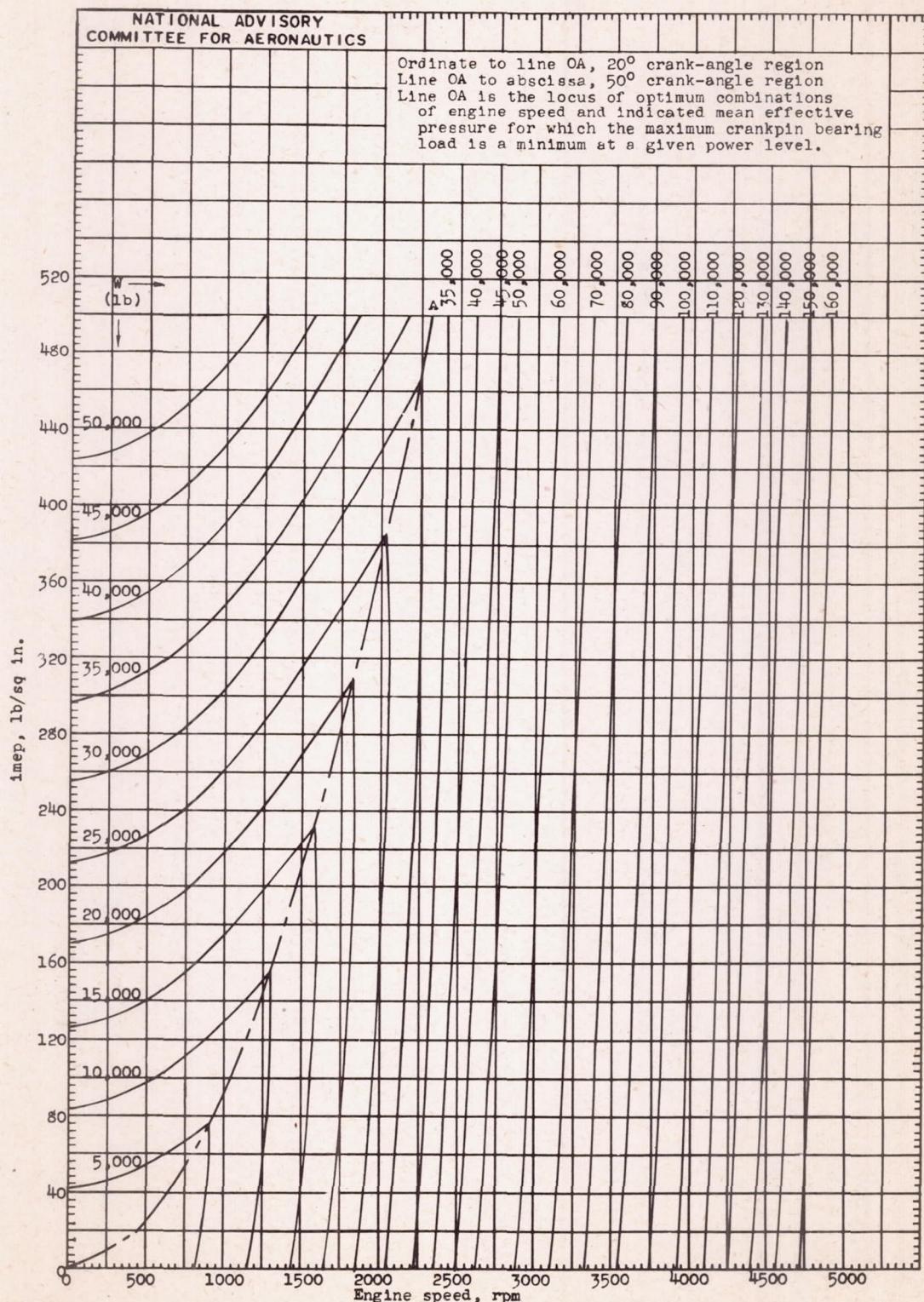
Figure 8. - Maximum load on crankpin of production engine A at a compression ratio of 6.7 for values of indicated mean effective pressure from 0 to 500 pounds per square inch and values of engine speed from 0 to 5000 rpm. Effective bearing area, 10.1 square inches. (For application of this chart to other radial engines having nine cylinders per crankpin see APPLICATION OF THE DIMENSIONAL ANALYSIS METHOD TO OTHER ENGINES HAVING NINE CYLINDERS PER CRANKPIN.)
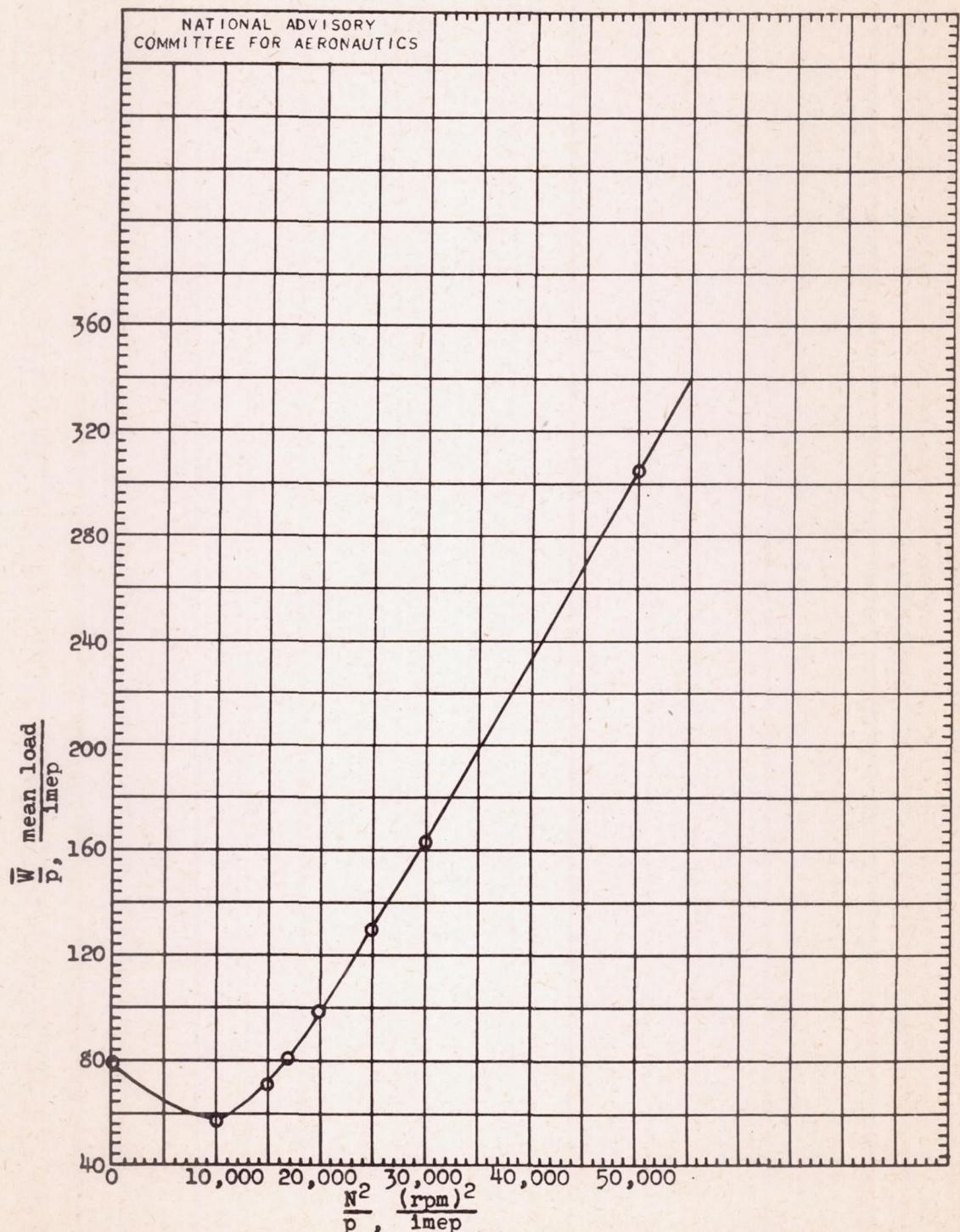
Figure 9. – Variation of $\overline{W}/p$ with N^2/p for engine A at a compression ratio of 6.7. (For application of this chart to other radial engines having nine cylinders per crankpin see APPLICATION OF THE DIMENSIONAL ANALYSIS METHOD TO OTHER ENGINES HAVING NINE CYLINDERS PER CRANKPIN.)
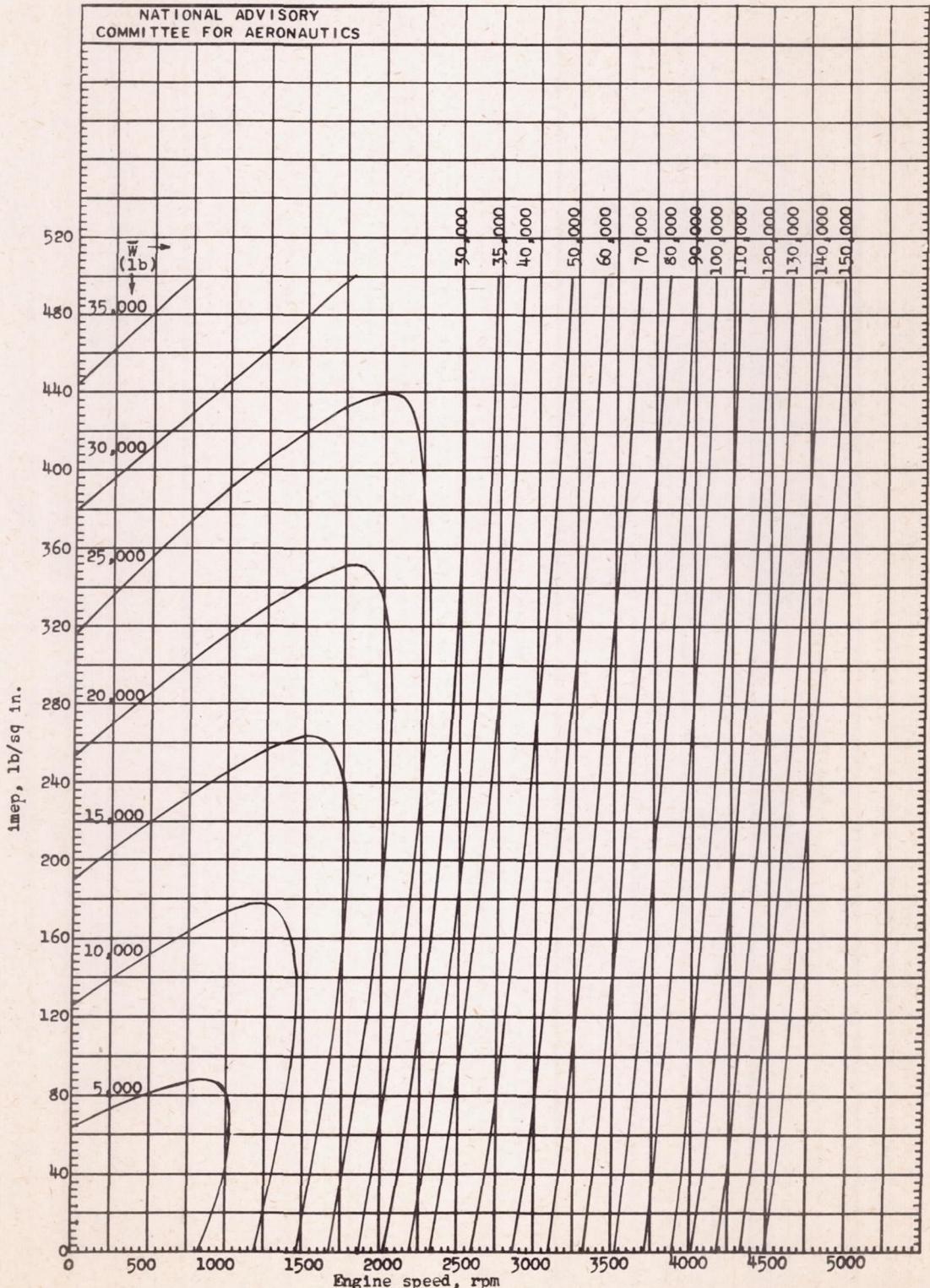
Figure 10. - Mean load on the crankpin of production engine A at a compression ratio of 6.7 for values of indicated mean effective pressure from 0 to 500 pounds per square inch and values of engine speed from 0 to 5000 rpm. Effective bearing area, 10.1 square inches. (For application of this chart to other radial engines having nine cylinders per crankpin see APPLICATION OF THE DIMENSIONAL ANALYSIS METHOD TO OTHER ENGINES HAVING NINE CYLINDERS PER CRANKPIN.)

imep (lb/sq in.)	Mean load (lb)
0	45,200
150	36,500
250	32,000
350	30,000

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Figure 11. - Polar diagrams showing the magnitude of the re-
sultant force on the crankpin of engine A and its direction
with respect to the engine axis at an engine speed of 2500
rpm and various indicated mean effective pressures for a
compression ratio of 6.7.
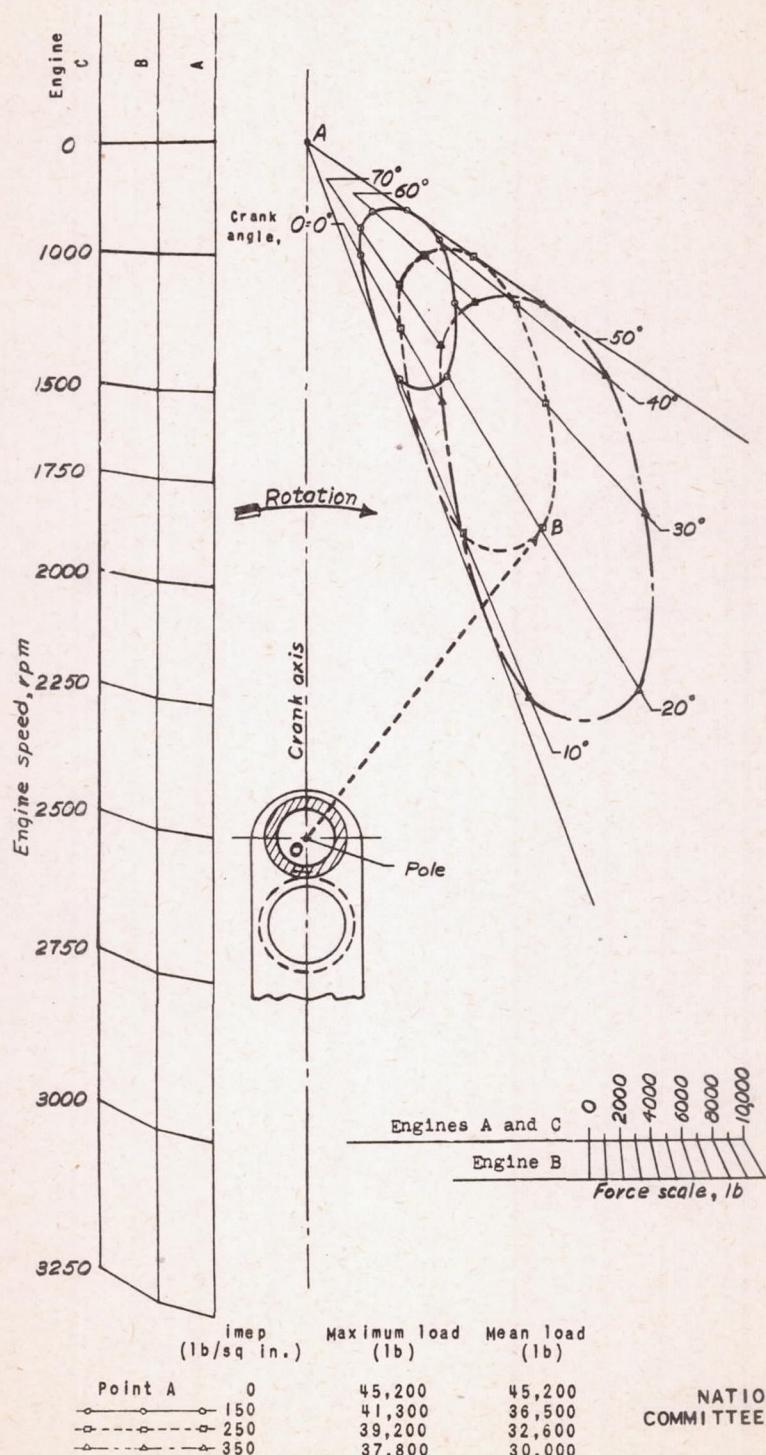
Figure 12. - Polar diagrams showing the magnitude of the resultant force on the crankpins of production engines A, B, and C, and its direction with respect to the crank axis at various engine speeds and indicated mean effective pressures for a compression ratio of 6.7.

	imep (lb/sq in.)	Maximum load (lb)	Mean load (lb)
Point A	0	45,200	45,200
	150	41,300	36,500
	250	39,200	32,600
	350	37,800	30,000

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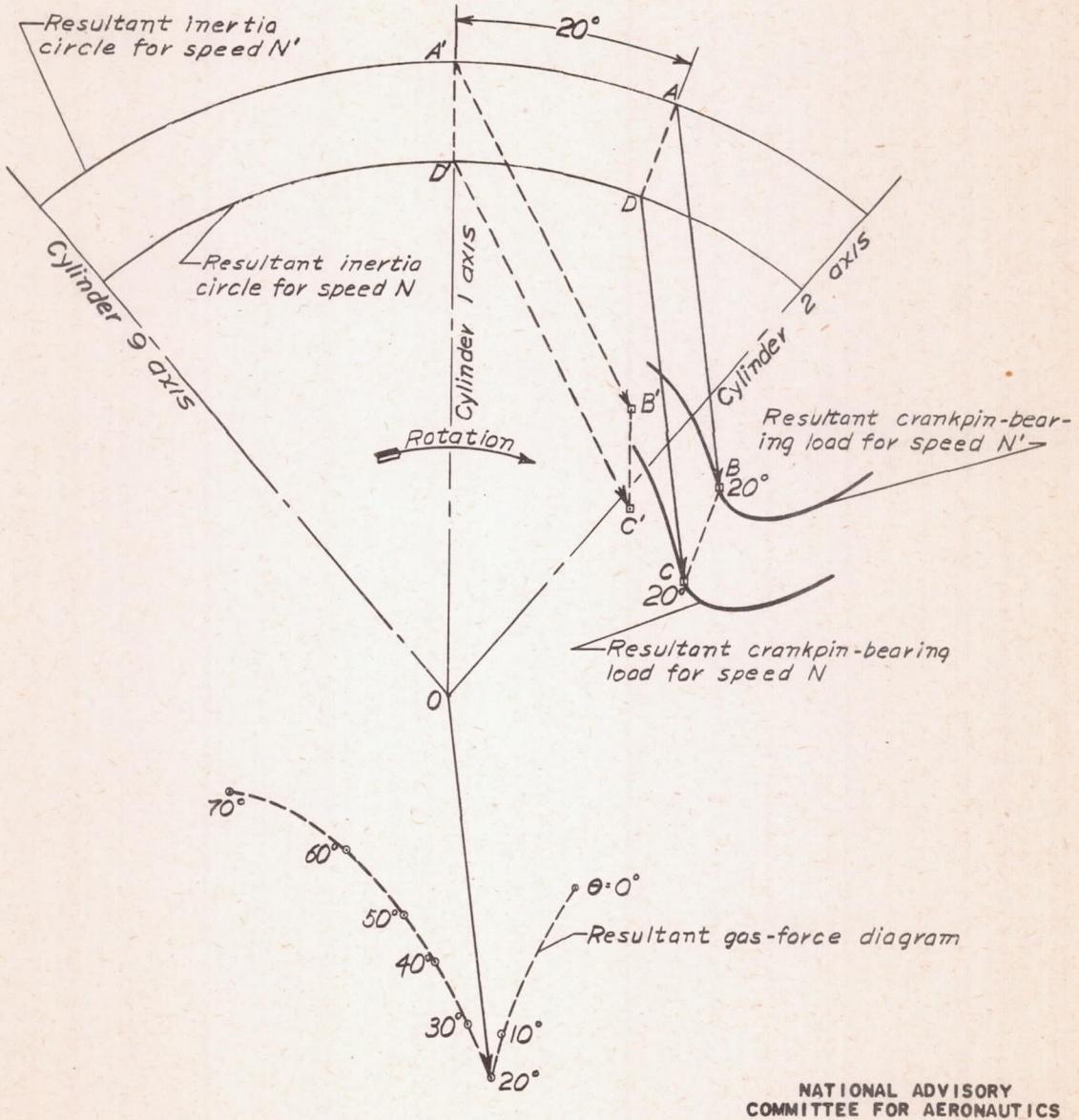
Figure 13. - Graphical verification to show that the polar diagram with respect to the crank axis is translated in a direction parallel to this axis when the engine speed varies and the indicated mean effective pressure is kept constant.

Fig. 14
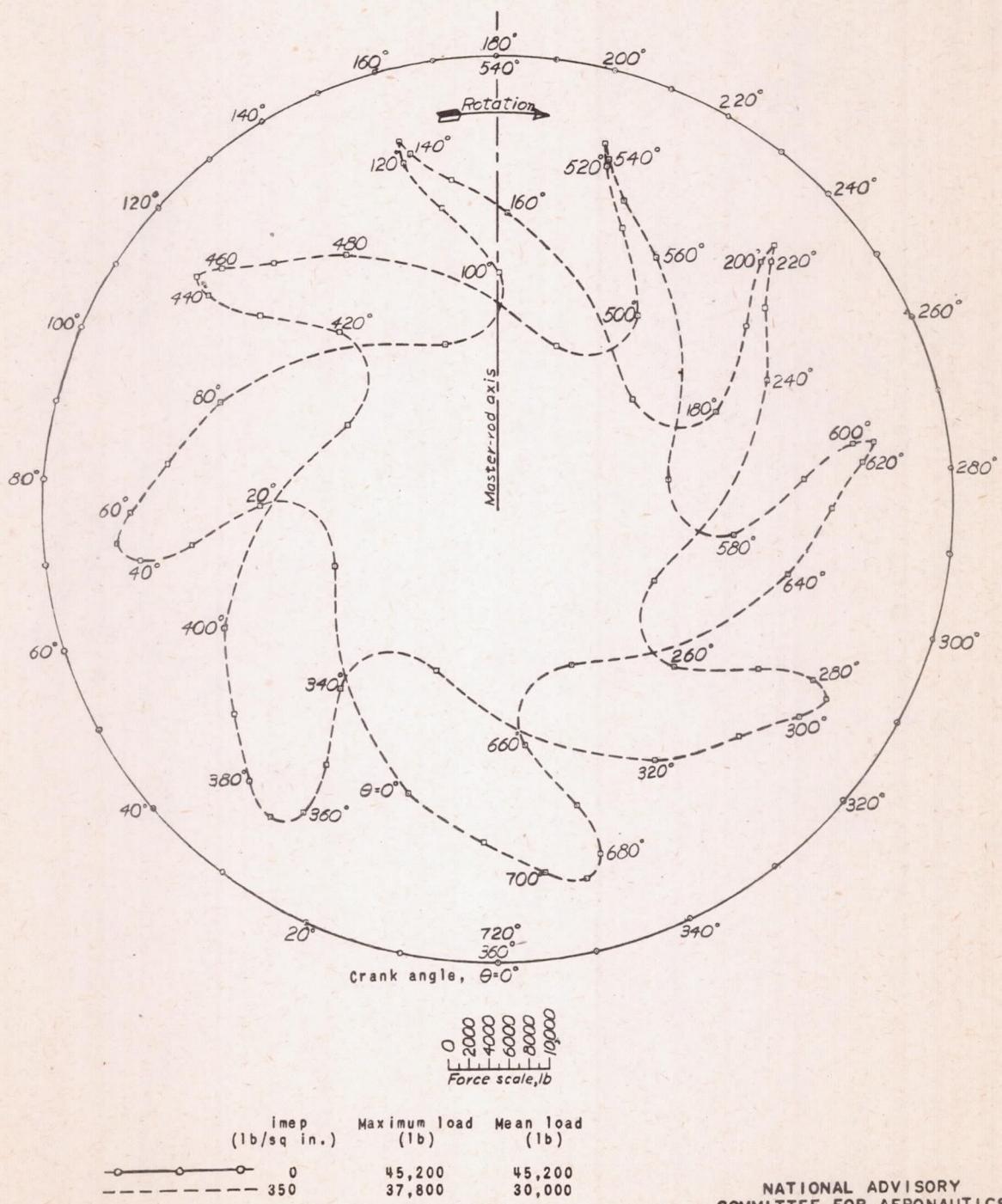
Figure 14. - Polar diagram showing the magnitude of the resultant force on the crankpin bearing of production engine A and its direction with respect to the master connecting rod axis at an engine speed of 2500 rpm, indicated mean effective pressures of 0 and 350 pounds per square inch, and a compression ratio of 6.7.
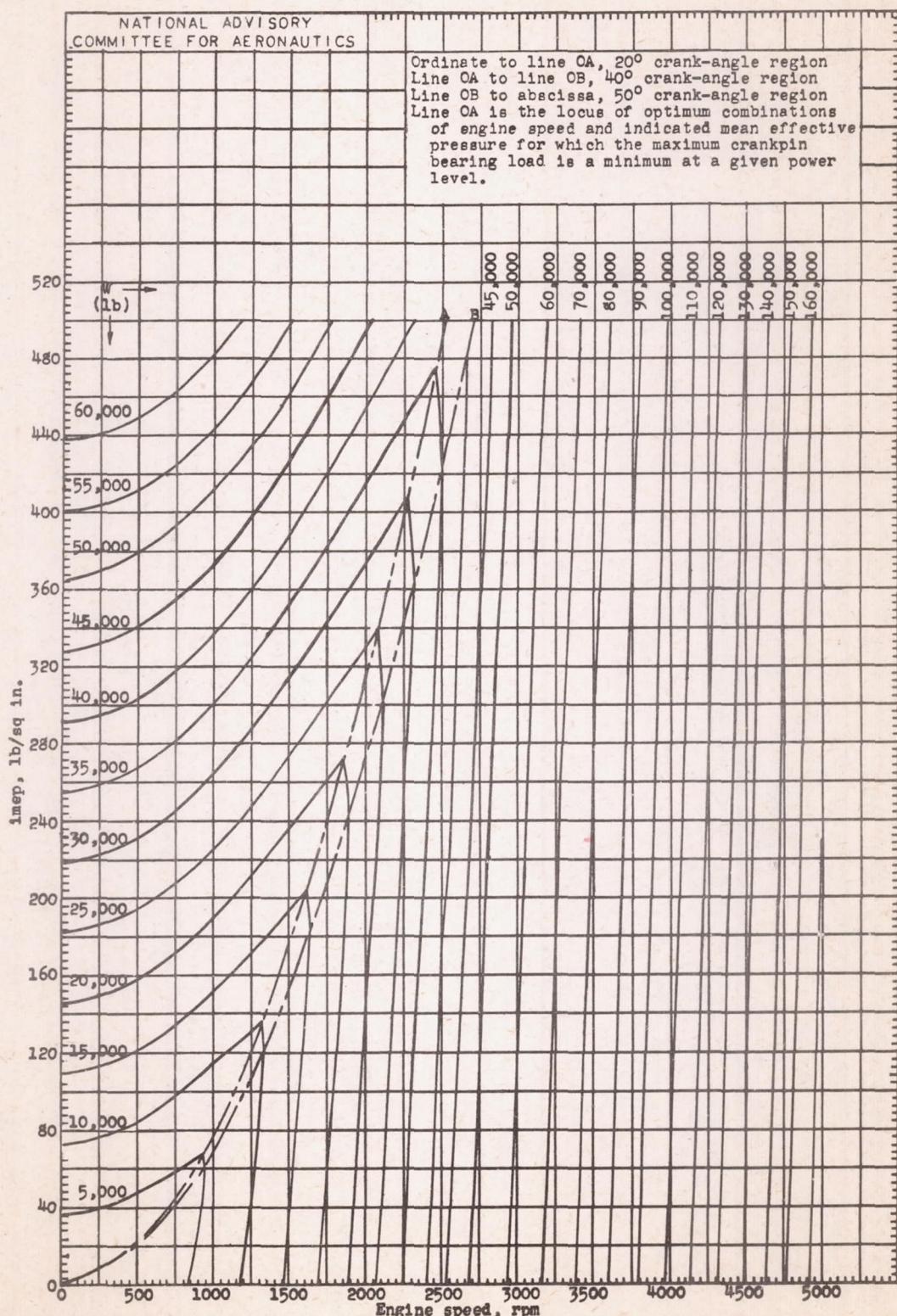
(a) Compression ratio, 8.0.

Figure 15. – Maximum load on crankpin of production engine A at two compression ratios for values of indicated mean effective pressure from 0 to 500 pounds per square inch and values of engine speed from 0 to 5000 rpm. Effective bearing area, 10.1 square inches. (For application of this chart to other radial engines having nine cylinders per crankpin see APPLICATION OF THE DIMENSIONAL ANALYSIS METHOD TO OTHER ENGINES HAVING NINE CYLINDERS PER CRANKPIN.)

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Ordinate to line OA, 20° crank-angle region
Line OA to abscissa, 50° crank-angle region
Line OA is the locus of optimum combinations
of engine speed and indicated mean effective
pressure for which the maximum crankpin bearing
load is a minimum at a given power level.

(b) Compression ratio, 9.0.
Figure 15. - Concluded.
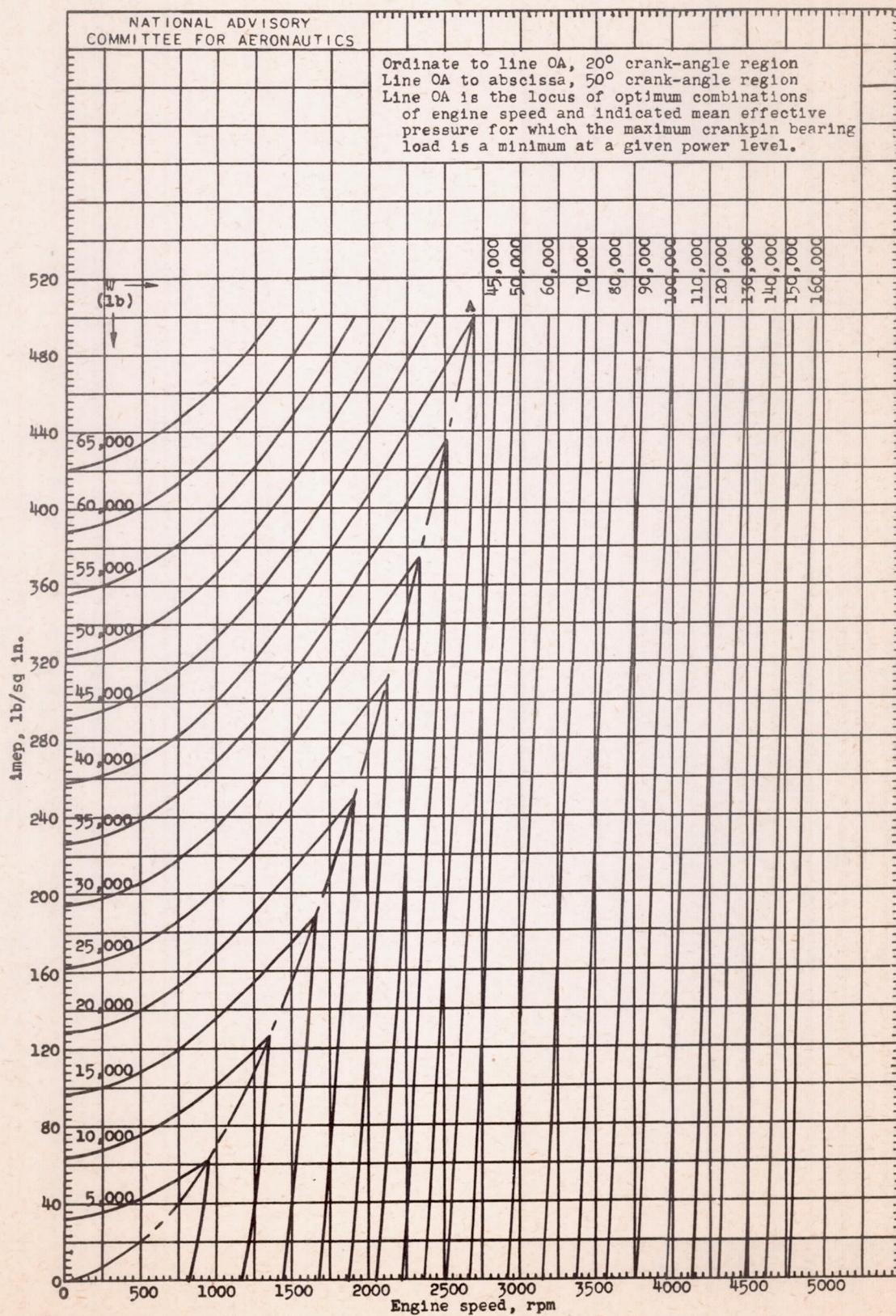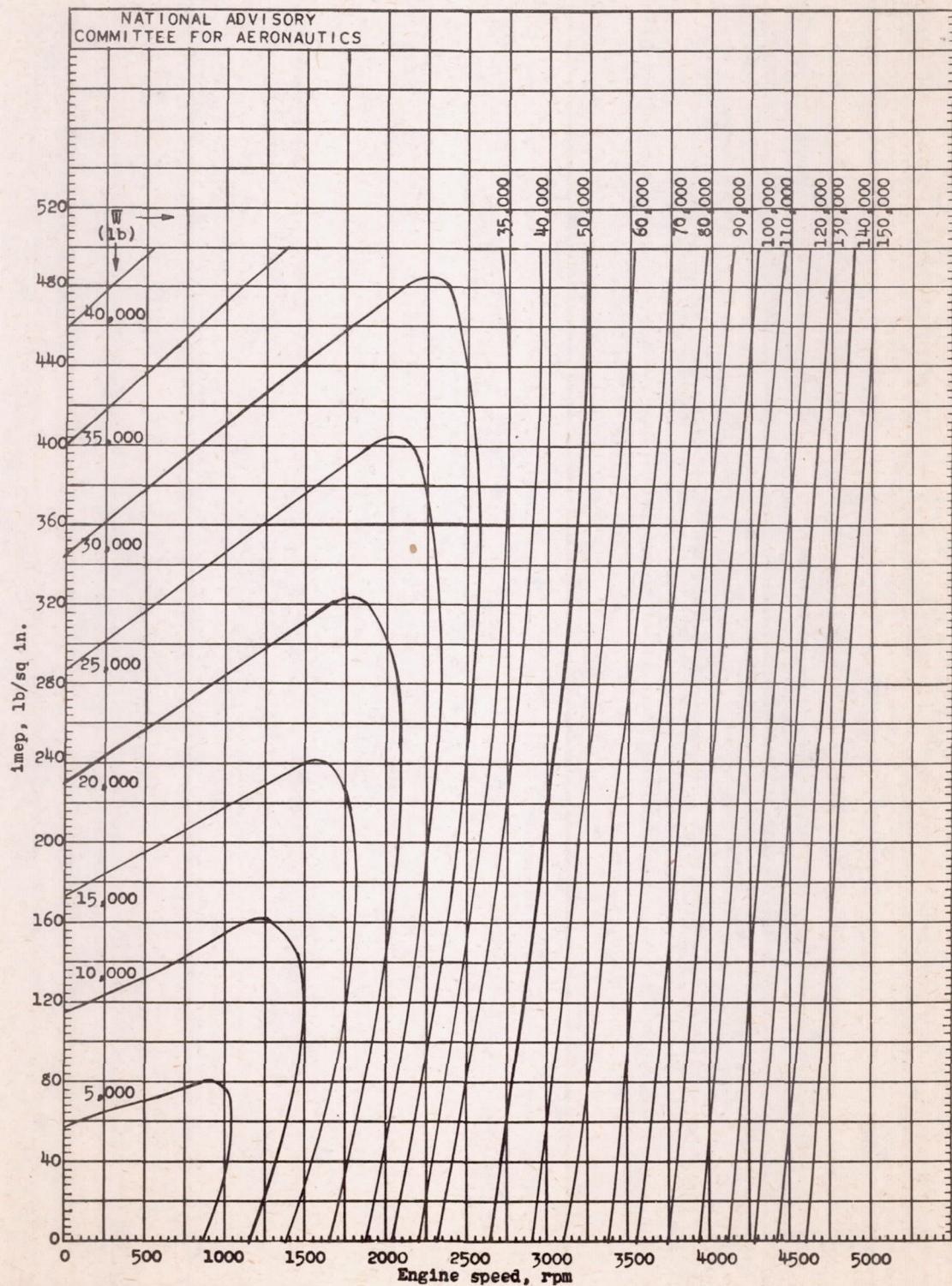
(a) Compression ratio, 8.0.

Figure 16. - Mean load on crankpin of production engine A at two compression ratios for values
of indicated mean effective pressure from 0 to 500 pounds per square inch and values of en-
gine speed from 0 to 5000 rpm. Effective bearing area, 10.1 square inches. (For application
of this chart to other radial engines having nine cylinders per crankpin see APPLICATION OF
THE DIMENSIONAL ANALYSIS METHOD TO OTHER ENGINES HAVING NINE CYLINDERS PER CRANKPIN.)

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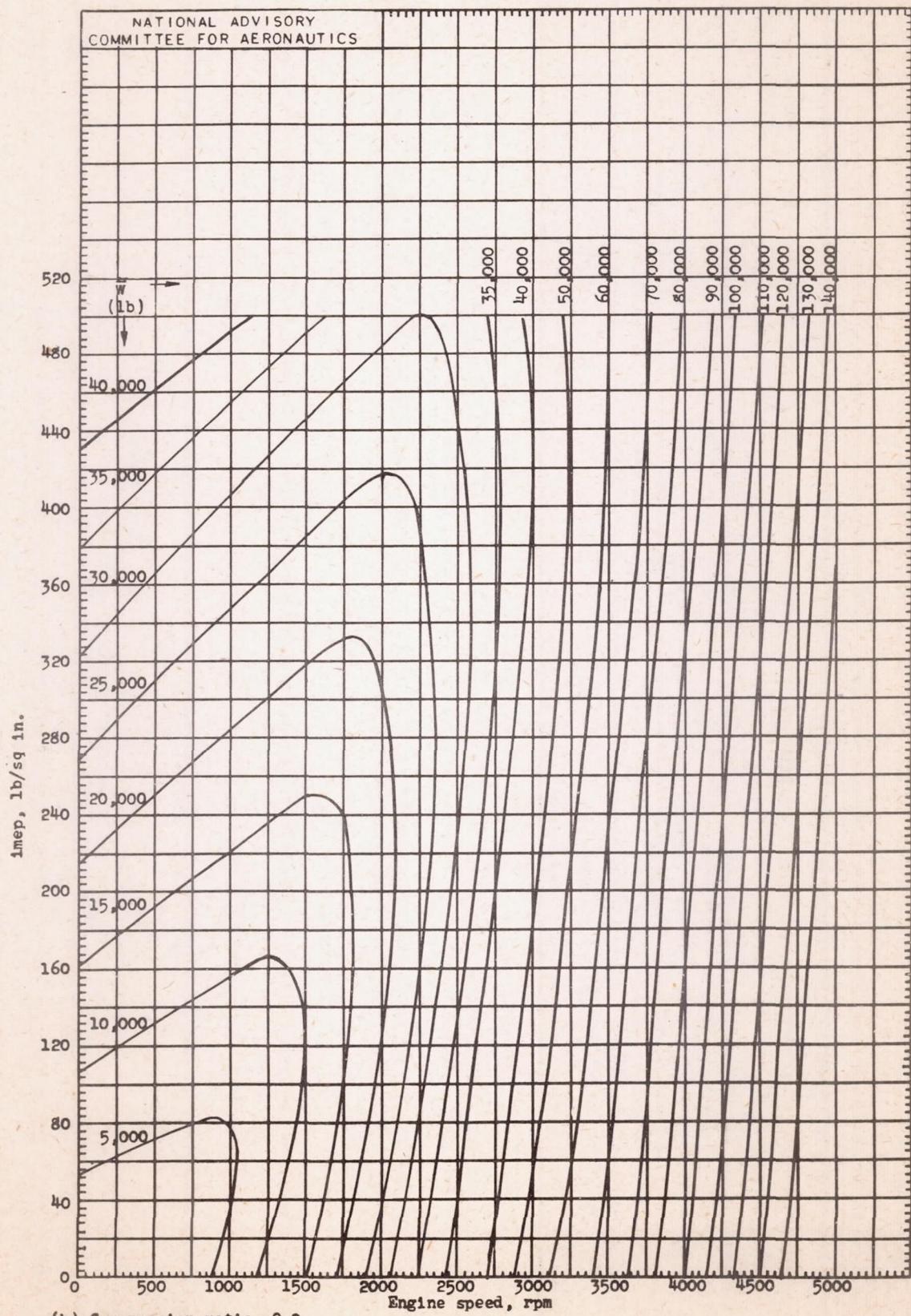
(b) Compression ratio, 9.0.
Figure 16. - Concluded.
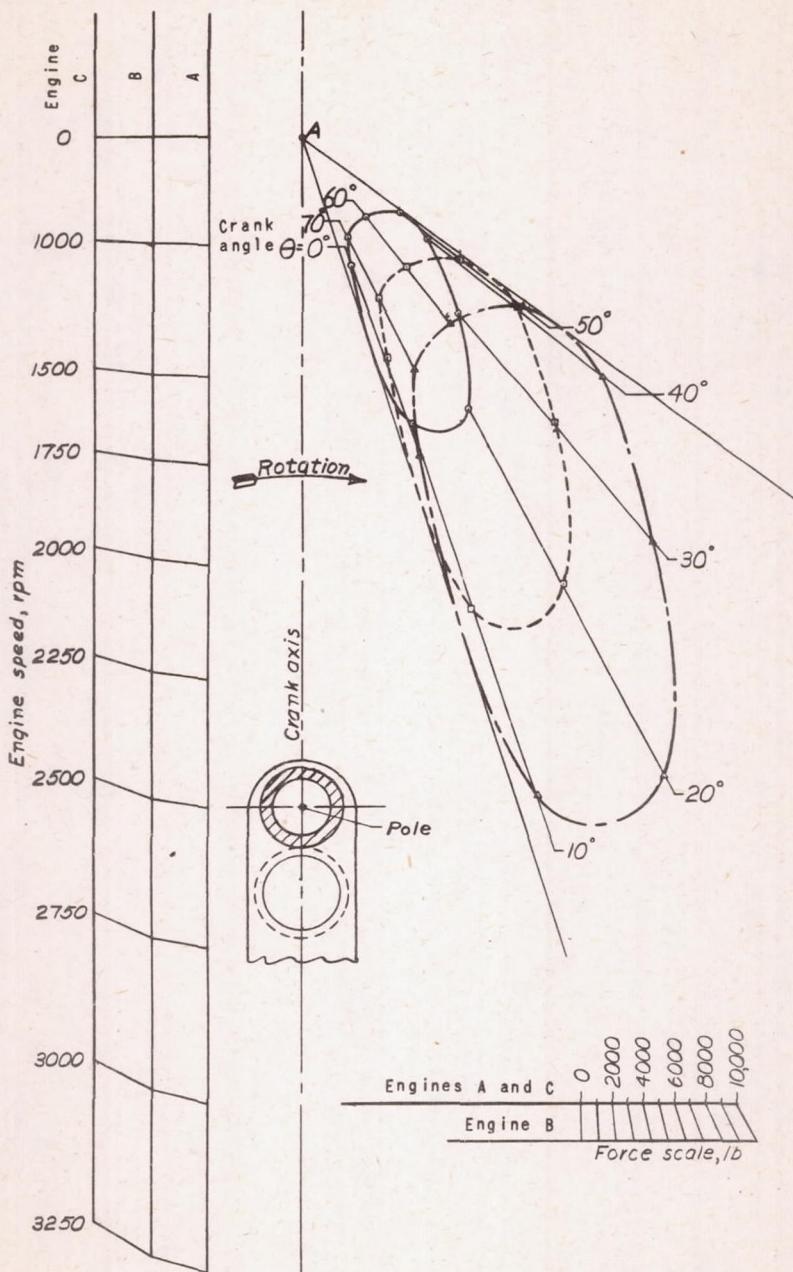
	imep (lb/sq in.)	Maximum load (lb)	Mean load (lb)
Point A	0	45,200	45,200
○————○————○	150	40,800	36,600
○————○————○	250	38,300	30,000
△—·—△—·—△	350	36,000	28,800

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(a) Compression ratio, 8.0.

Figure 17. - Polar diagrams showing the magnitude of the re-
sultant force on the crankpins of production engines A, B,
and C, and its direction with respect to the crank axis at
different engine speeds and indicated mean effective pres-
sures.

	imep (lb/sq in.)	Maximum load (lb)	Mean load (lb)
Point A	0	45,200	45,200
	150	40,300	34,000
	250	37,600	29,100
	350	35,200	28,400
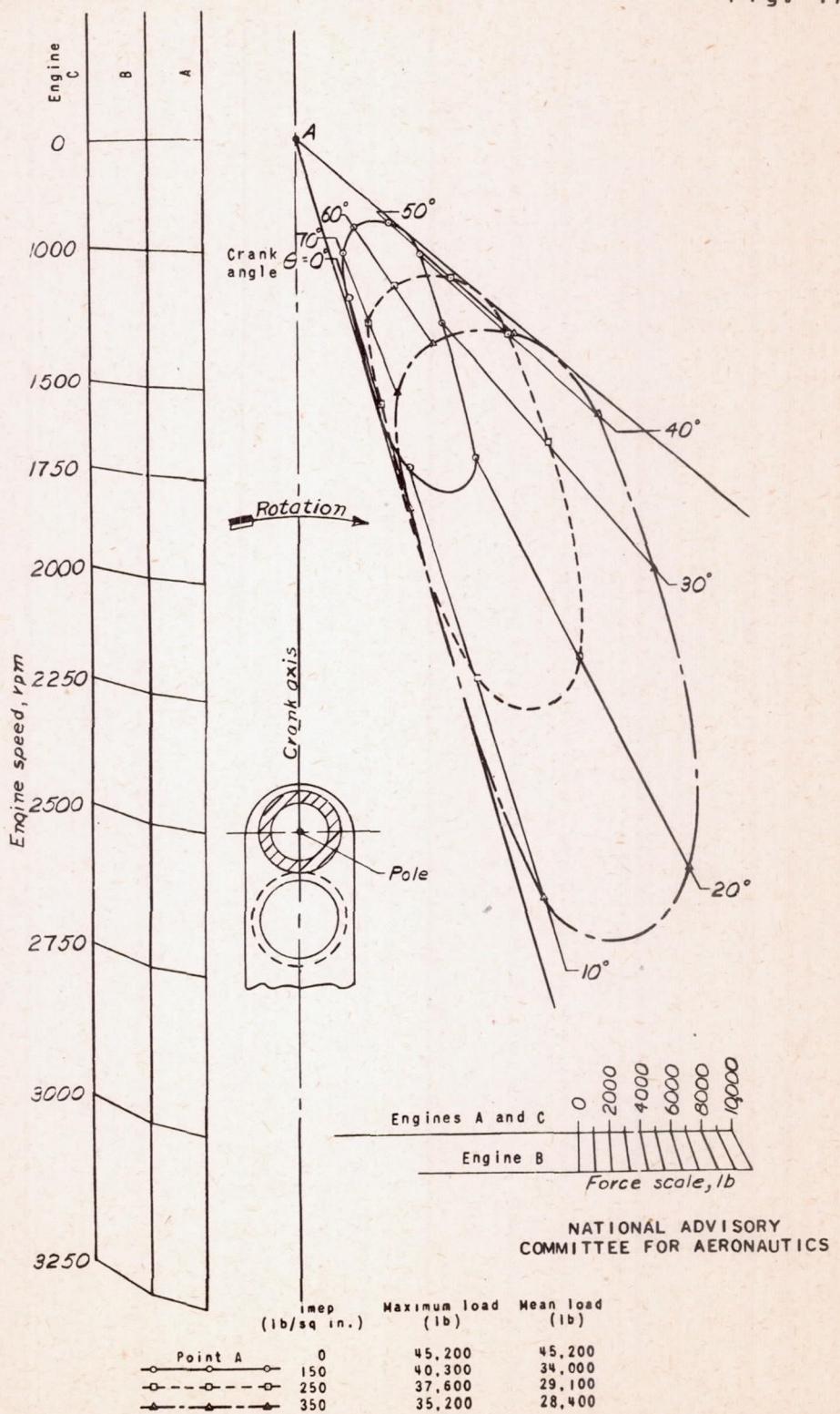
(b) Compression ratio, 9.0.

Figure 17. - Concluded.
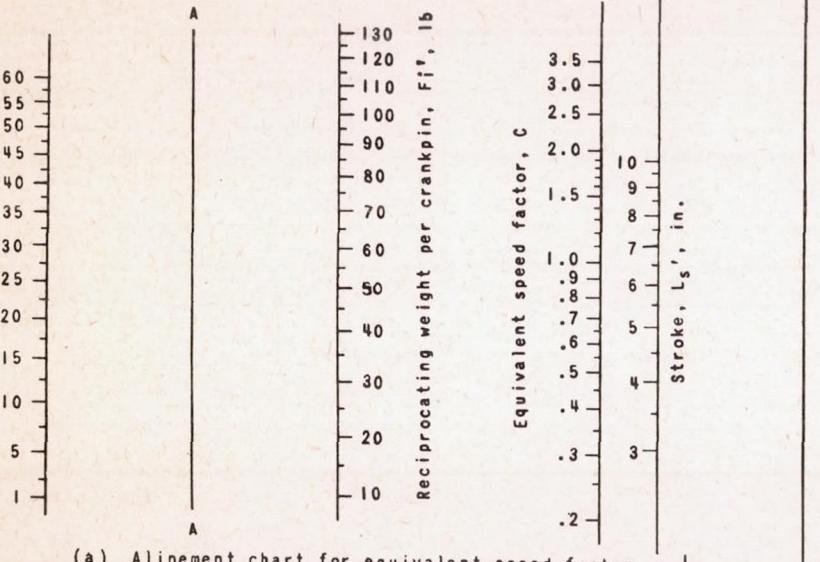
(a) Alinement chart for equivalent speed factor.

$$C = 0.0315 \left[\frac{L_s'(32.72 + 0.516\ F_i')(F_c' + 41.50)}{D'} \right]^{\frac{1}{2}}$$

Connect F_c' with F_i', obtain intersection on A;
connect L_s' with D', obtain intersection on B;
connect A with B and obtain intersection on C.
Example: F_c', 32.72; F_i', 80.4; L_s', 6.875; D'; 6.125.
Answer: C, 1.00.
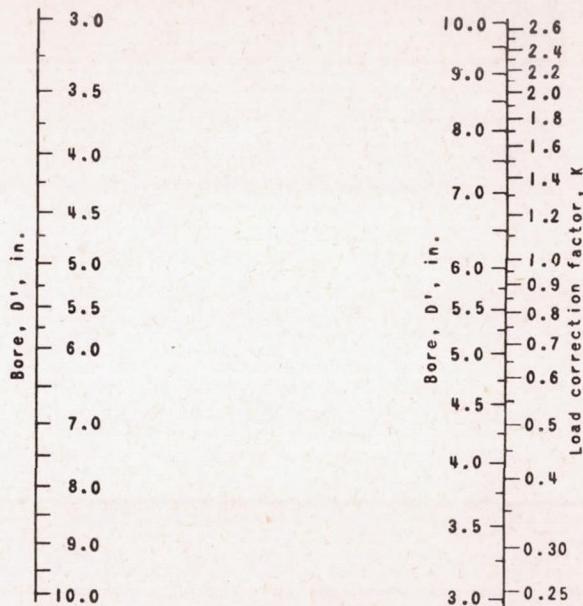
(b) Alinement chart for load correction factor.
$K = 0.0266(D')^2$
Example: $D' = 6.125$
Answer: K, 1.00.

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Figure 18. - Alinement charts for equivalent speed and load correction factors.

Fig. 18
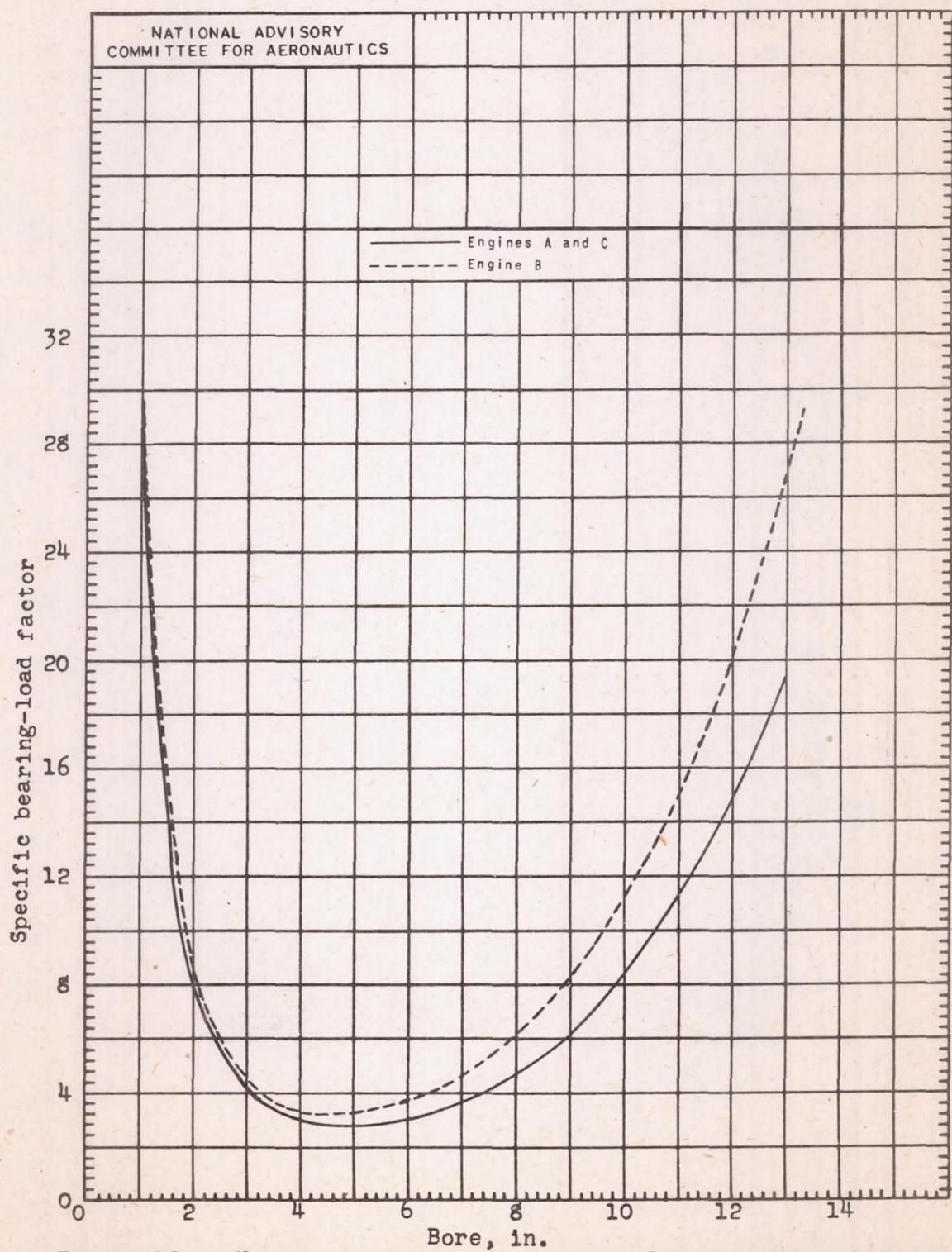
Figure 19. - Variation of specific bearing-load factor with bore.